UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE MAT1A2E

CAMPUS APK

ASSESSMENT SUPPLEMENTARY EXAM

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| INTERNAL MODERATOR | MR M SIAS |
| DURATION 2 HOURS | MARKS 75 |

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 15 PAGES, INCLUDING COVER PAGE

INSTRUCTIONS: 1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [15 marks]

For questions 1.1 – 1.15, choose **one** correct answer, and mark with an (X) in the correct block.

| Question | **a** | **b** | **c** | **d** | **e** | CORRECTION |
|---|---|---|---|---|---|---|
| 1.1 | | | | | | |
| 1.2 | | | | | | |
| 1.3 | | | | | | |
| 1.4 | | | | | | |
| 1.5 | | | | | | |
| 1.6 | | | | | | |
| 1.7 | | | | | | |
| 1.8 | | | | | | |
| 1.9 | | | | | | |
| 1.10 | | | | | | |
| 1.11 | | | | | | |
| 1.12 | | | | | | |
| 1.13 | | | | | | |
| 1.14 | | | | | | |
| 1.15 | | | | | | |

1.1) Which one of the following is a negation of

"Tim is inside and Leo is at the pool."

(1)

a) Tim is inside or Leo is not at the pool.

b) Tim is inside or Leo is at the pool.

c) Tim is not inside or Leo is at the pool.

d) Tim is not inside and Leo is not at the pool.

e) Tim is not inside or Leo is not at the pool.

1.2) Which one of the following is a tautology? (1)

a) $B \wedge \neg B$

b) $\neg A \vee \neg B$

c) $\neg(\neg A \wedge A)$

d) $A \rightarrow (B \wedge C)$

e) None of the above.

1.3) The proposition $p \wedge \neg q \rightarrow r$ is logically equivalent to [1]

a) $p \wedge (q \vee r)$

b) $\neg p \vee (q \vee r)$

c) $(p \wedge q) \vee (p \wedge r)$

d) $\neg p \wedge (q \vee r)$

e) None of the above

1.4) Which of the following statements is the contrapositive of the statement: [1]

"You win the game if you know the rules but are not overconfident".

a) If you lose the game, then you don't know the rules or you are overconfident.

b) A sufficient condition that you win the game is that you know the rules or you are not overconfident.

c) If you don't know the rule or are overconfident, you lose the game.

d) If you know the rules and are overconfident, then you win the game.

e) None of the above

1.5) A sufficient condition that a triangle, $T$, be a right angle triangle is that $a^2 + b^2 = c^2$. An equivalent statement is: [1]

a) If $T$ is a right angle triangle, then $a^2 + b^2 = c^2$.

b) If $a^2 + b^2 = c^2$, then $T$ is a right angle triangle.

c) If $a^2 + b^2 \neq c^2$, then $T$ is not a right angle triangle.

d) $T$ is a right angle triangle only if $a^2 + b^2 = c^2$.

e) None of the above

1.6) The symbolization of the conjuction is [1]

a) $\neg p$.

b) $p \wedge q$.

c) $p \rightarrow q$.

d) $p \vee q$.

e) None of the above

1.7) The solution to $\lim_{x\to 2} \frac{x^2-4}{x-2}$ is: (1)

a) 0

b) $\infty$

c) 4

d) $-\infty$

e) None of the above

1.8) Evaluate the limit, if it exists. $\lim_{h\to 0} \frac{(x-h)^3 - x^3}{h}$. [1]

a) 1

b) $-3$

c) $3x^2$

d) $-3x^2$

e) None of the above

1.9) Use logarithmic differentiation to find the derivative of the function. $y = x^{6x}$. [1]

a) $y' = 6x^{6x}(6\ln x + 1)$

b) $y' = 6(\ln x + 1)$

c) $y' = 6x^{6x}(\ln x + 1)$

d) $y' = -6x^{6x}(6\ln x + 6)$

e) None of the above

1.10) Find $\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$. [1]

a) Does not exist.

b) $\frac{1}{2}$.

c) 0.

d) $-\frac{1}{2}$.

e) None of the above

1.11) Consider the curve defined below and select the correct description: [1]

$$y = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

a) $y$ is a function of $x$ and it is an increasing function

b) $y$ is a function of $x$ but it is not one-to-one

c) $y$ is not a function of $x$

d) $y$ is a function of $x$ and the function is one-to-one

e) None of the above

1.12) If $f(x) = x^3 - 1$ then $f^{-1}(26) =$ [1]

a) 0

b) 1

c) 2

d) 3

e) None of the above

1.13) Find the $\lim_{x \to 0} \dfrac{\tan(x)}{x}$. [1]

a) $\frac{1}{\pi}$.

b) 0.

c) 1.

d) $\pi$.

e) None of the above

1.14) $\dfrac{\sqrt{-12}}{\sqrt{-4}} = \ldots.$ [1]

a) $-\sqrt{3}$.

b) $-i\sqrt{3}$.

c) $i\sqrt{3}$.

d) $\sqrt{3}$.

e) None of the above

1.15) Evaluate the sum $i^1 + i^2 + \cdots + i^{1000} = \ldots$, where $i$ is a complex number. [1]

a) 1.

b) $-1$.

c) 0.

d) 1000.

e) None of the above

Question 2 [4 marks]

Consider $f(x) = \frac{1}{\sqrt{x}}$ and $g(x) = 9 - x^2$.

**(a)** Find the function $(f \circ g)(x)$. (2)

**(b)** Find the domain of $f \circ g$. (2)

Question 3 [2 marks]

Find the horizontal asymptotes of $f(x) = \dfrac{1}{x^2 - 1}$

Question 4 [3 marks]

Sketch the function $y = |e^x - 1|$. Mark all asymptotes and axis intercepts, if they exist.

Question 5 [1 mark]

Complete the statement:

Let $f$ be a function defined on an open interval that contains $a$. Then $\lim\limits_{x \to a} f(x) = L$ if for every $\epsilon > 0$ there is a $\delta > 0$ such that ...

Question 6 [19 marks]

Calculate the following limits:

a)

$$\lim_{x \to 3^-} \left( \frac{x}{x-3} \right)$$

(2)

b) $\lim_{x \to 0} f(x)$ where

$$f(x) = \begin{cases} x^2 + 2\cos x + 1 & if\ x < 0 \\ e^x - 4 & if\ x \geq 0 \end{cases}$$

(3)

c)

$$\lim_{x \to \left(\frac{\pi}{2}\right)^-} \ln(\cos(x))$$

. (4)

d)

$$\lim_{x\to-\infty} \frac{\sqrt{8x^2+1}}{3x+2}.$$

(4)

e)

$$\lim_{x\to\infty} \frac{\sqrt{8x^2+1}}{3x+2}.$$

(2)

f)

$$\lim_{x\to 0}\sqrt{\frac{\sin x}{x}}.$$

(4)

Question 7 [3 marks]

If $3x \leqslant g(x) \leqslant x^3 + 2$ on the interval $[0, 2]$, evaluate $\lim_{x\to 1} g(x)$.

Question 8 [4 marks]

Given: $f(x) = \dfrac{x^2 - 25}{2x - 6}$.

Find:

a) Vertical asymptote/s (2)

b) Horizontal asymptote/s (2)

Question 9 [3 marks]

The radius of a circular puddle is growing at a rate of $5cm/sec$. How fast is the area growing at the instant when the radius is $10cm$?

Question 10 [4 marks]

Find the $127^{th}$ derivative of $\sin(x)$.

Question 11 [3 marks]

Differentiate the function $f(x) = \dfrac{\tan(mx)}{x}$.

Question 12 [10 marks]

a) Rewrite the following statement in the language of First Order Logic: (2)

**No computer scientists are unemployed.**

b) Construct a truth table for $p \vee (\neg p \wedge q) \rightarrow \neg q$ (4)

c) Prove by Mathematical Induction (4)

$$4^3 + 4^4 + 4^5 + \cdots + 4^n = \frac{4(4^n - 16)}{3}, \qquad n \geq 3.$$

Question 13 [4 marks]

a) Simplify $(9 + 8i) - 3(2 + 8i) + (9 + 2i)$ (2)

b) Write in the form $a + bi$. (2)

$$\frac{4 + 8i}{-6i}$$