UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE MAT1A2E

CAMPUS APK

ASSESSMENT NOVEMBER EXAM

DATE 16/11/2019 TIME 12:30

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DURATION 2 HOURS MARKS 75

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 15 PAGES, INCLUDING COVER PAGE

INSTRUCTIONS: 1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [15 marks]

For questions 1.1 – 1.15, choose **one** correct answer, and mark with an (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**	CORRECTION
1.1						
1.2						
1.3						
1.4						
1.5						
1.6						
1.7						
1.8						
1.9						
1.10						
1.11						
1.12						
1.13						
1.14						
1.15						

1.1) Which one of the following is a negation of

"Tim is inside and Leo is at the pool."

(1)

a) Tim is inside or Leo is not at the pool.

b) Tim is inside or Leo is at the pool.

c) Tim is not inside or Leo is at the pool.

d) Tim is not inside and Leo is not at the pool.

e) Tim is not inside or Leo is not at the pool.

1.2) Which one of the following is a tautology? (1)

a) $B \wedge \neg B$

b) $\neg A \vee \neg B$

c) $\neg(\neg A \wedge A)$

d) $A \rightarrow (B \wedge C)$

e) None of the above.

1.3) The proposition $p \wedge \neg q \rightarrow r$ is logically equivalent to [1]

a) $p \wedge (q \vee r)$

b) $\neg p \vee (q \vee r)$

c) $(p \wedge q) \vee (p \wedge r)$

d) $\neg p \wedge (q \vee r)$

e) None of the above

1.4) Which of the following statements is the contrapositive of the statement: [1]

"You win the game if you know the rules but are not overconfident".

a) If you loose the game, then you don't know the rules or you are overconfident.

b) A sufficient condition that you win the game is that you know the rules or you are not overconfident.

c) If you don't know the rule or are overconfident, you loose the game.

d) If you know the rules and are overconfident, then you win the game.

e) None of the above

1.5) A sufficient condition that a triangle, T, be a right angle triangle is that $a^2 + b^2 = c^2$. An equivalent statement is: [1]

a) If T is a right angle triangle, then $a^2 + b^2 = c^2$.

b) If $a^2 + b^2 = c^2$, then T is a right angle triangle.

c) If $a^2 + b^2 \neq c^2$, then T is not a right angle triangle.

d) T is a right angle triangle only if $a^2 + b^2 = c^2$.

e) None of the above

1.6) The symbolization of the conjuction is [1]

a) $\neg p$.

b) $p \wedge q$.

c) $p \rightarrow q$.

d) $p \vee q$.

e) None of the above

1.7) The solution to $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$ is: (1)

a) 0

b) ∞

c) 4

d) $-\infty$

e) None of the above

1.8) Consider the curve defined below and select the correct description: [1]

$$y = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

a) y is a function of x and it is an increasing function

b) y is a function of x but it is not one-to-one

c) y is not a function of x

d) y is a function of x and the function is one-to-one

e) None of the above

1.9) If $f(x) = x^3 - 1$ then $f^{-1}(26) =$ [1]

a) 0

b) 1

c) 2

d) 3

e) None of the above

1.10) Evaluate the limit, if it exists. $\lim_{h\to 0} \dfrac{(x-h)^3 - x^3}{h}$. [1]

a) 1

b) -3

c) $3x^2$

d) $-3x^2$

e) None of the above

1.11) Use logarithmic differentiation to find the derivative of the function. $y = x^{6x}$. [1]

a) $y' = 6x^{6x}(6\ln x + 1)$

b) $y' = 6(\ln x + 1)$

c) $y' = 6x^{6x}(\ln x + 1)$

d) $y' = -6x^{6x}(6\ln x + 6)$

e) None of the above

1.12) Evaluate the sum $i^1 + i^2 + \cdots + i^{1000} = \ldots$, where i is a complex number. [1]

a) 1.

b) -1.

c) 0.

d) 1000.

e) None of the above

1.13) $\dfrac{\sqrt{-12}}{\sqrt{-4}} = \ldots$. [1]

a) $-\sqrt{3}$.

b) $-i\sqrt{3}$.

c) $i\sqrt{3}$.

d) $\sqrt{3}$.

e) None of the above

1.14) Find $\lim\limits_{x\to 0} \dfrac{e^x - x - 1}{x^2}$. [1]

a) Does not exist.

b) $\frac{1}{2}$.

c) 0.

d) $-\frac{1}{2}$.

e) None of the above

1.15) Find the $\lim_{x\to 0} \dfrac{\tan(x)}{x}$. [1]

a) $\frac{1}{\pi}$.

b) 0.

c) 1.

d) π.

e) None of the above

Question 2 [2 marks]

Determine whether $f(x)$ is even, odd or neither: $f(x) = x^3 - x^2 - x$.

Question 3 [1 mark]

The x-intercept of $g(x)$ is 3. What is the x-intercept of $g\left(\dfrac{x}{4}\right)$?

Question 4 [2 marks]

Given $f(x) = \sqrt[3]{4x^7 - 1}$. Determine an equation for $f^{-1}(x)$

Question 5 [3 marks]

Given:

$$f(x) = \begin{cases} |x| & \text{if } x < 2 \\ x - 3 & \text{if } x \geq 2 \end{cases}$$

a) Sketch the graph of $f(x)$. (2)

b) Calculate $\lim_{x \to 0} f(x)$ (1)

Question 6 [2 marks]

Use transformations to draw the graph of $y = -\frac{1}{2}\sqrt{2 + x}$. Clearly show the x- and y-intercepts.

Question 7 [3 marks]

Given $\varepsilon > 0$, show how to find a δ so that you can prove $\lim_{x \to 4} \left(\frac{3x}{2} + 5 \right) = 11$. **(You do not need to prove that your δ works.)**

Question 8 [4 marks]

Calculate the following limits:

a) $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$. (2)

b) $\lim_{x \to 1} \dfrac{x^3 - 1}{x^2 - 1}$ (2)

Question 9 [3 marks]

Use the Squeeze Theorem to evaluate $\lim_{x \to \infty} \dfrac{2\cos(x)}{x^2 + 3}$.

Question 10 [9 marks]

a) Rewrite the following statement in the language of first order logic: (2)

Every real number is negative, zero or positive.

b) Construct a truth table for $p \vee (\neg p \wedge q) \rightarrow q$. (4)

c) Prove that if n is an integer, then $3n^2 + n + 14$ is even. (3)
(**Use the method of proof by cases.**)

Question 11 [4 marks]

Consider the function $f(x)$ where

$$f(x) = \begin{cases} 5 - x & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 5 \\ (x-5)^2 & \text{if } x > 5 \end{cases}.$$

Determine the values of a for which $\lim\limits_{x \to a} f(x)$ exists.

Question 12 [3 marks]

Find $D_x[e^{4\cosh(\sqrt{x})}]$.

Question 13 [4 marks]

a) Simplify $9i(9-8i)-(2+8i)+(9+2i)$ (2)

b) Write in the form $a+bi$: $\dfrac{2}{5-3i}$ (2)

Question 14 [16 marks]

Evaluate the following limits.

a) (4)

$$\lim_{x\to 0^-} \arctan\left(\frac{1}{x}\right).$$

b) $\lim_{x\to-\infty} \frac{\sqrt{3x^2-1}}{5-x}$. (4)

c) $\lim_{x\to\infty} \frac{\sqrt{3x^2-1}}{5-x}$. (2)

d) $\lim_{x \to 0} \frac{\sin(3x)}{7x}$. (4)

Question 15 [4 marks]

Find the 127^{th} derivative of $\sin(x)$.

Question 16 [3 marks]

Differentiate the function $f(x) = \dfrac{\cos(mx)}{x}$.