UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

Department of Pure and Applied Mathematics

Module MAT1DB1
Bio and Enviro Maths and Stats

Campus: APK
Assessment: Supplementary Summative Assessment

Date: January 2020	Time: TBA
Assessor	Andrew Einhorn
Internal Moderator	Miles Askes
Duration 120 minutes	Marks 85
Number of pages: 1-11 pages	

FIRST + LAST NAME ________________________________

STUDENT NUMBER ________________________________

CONTACT NUMBER ________________________________

TUTORIAL GROUP ________________________________

INSTRUCTIONS

1. **Answer all of the questions on the paper in pen. You may use pencils to sketch graphs or draw diagrams.**
2. **Calculators may be used on this test. A z-table and t-test table are provided at the back of this paper.**
3. **Show all calculations and motivate all answers.**
4. **If you require extra space, continue your work on the <u>adjacent</u> blank page and indicate this clearly.**
5. **Unless stated otherwise, round all decimal answers to 3 significant figures.**

QUESTION 1 [11 marks]

Consider the function $f(x) = x^5 - 5x$

a. Find the x and y intercepts of this function:

(2)

b. Compute $f'(x)$ and use this to find any local minima or maxima of the function (both x- and y- values).

(3)

c. Compute $f''(x)$ and use this to find any points of inflection (x-values only)

(3)

d. Use this information to sketch $f(x)$ indicating any x- or y-intercepts, any minima/maxima and any inflection points.

(3)

QUESTION 2 [14 marks]

Integrate the following functions:

a. $\int \frac{3\sin(\ln\theta)}{\theta} d\theta$

(3)

b. $\int \ln(\mathrm{x}) dx$

(3)

c. $\int t^2 e^{-t} dt$

(4)

d. $\int \frac{x-7}{x^2-7x+12} dx$

(4)

QUESTION 3 [10 marks]

Compute the following definite integrals. If you use u-substitution, change the limits of your integral to reflect the change of variables.

a. $\int_0^1 5x - 5^x \quad dx$

(3)

b. $\int_0^1 \sin\left(\frac{\pi}{3}t\right) dt$

(3)

c. $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\, dt$

(4)

QUESTION 4 [10 marks]

Mr Mbatha enjoys drinking whiskey, but he has heard that because alcohol has a lower boiling point than water, the alcohol evaporates from his glass over time. Assuming the concentration of the alcohol, C, decreases at a rate proportional to itself:

a. Write a differential equation that describes the given information

(1)

b. Use separation of variables to solve the equation above and find an explicit formula for the concentration of the alcohol, C, as a function of time t (in minutes). Assuming the initial concentration of alcohol is 44%, solve for A.

(3)

c. Mr Mbatha uses an alcoholmeter to measure the concentration of alcohol after 12 hours and discovers it has fallen to 32%. Use this information to solve for the variable k (to 4 d.p).

(2)

d. How long will it take before the alcohol concentration falls below 10% (to the closest hour)?

(2)

e. Find C'(0) (to 2d.p) and give it the correct units?

(2)

QUESTION 5 [11 marks]

A rumour is spreading around campus that Black Coffee is going to perform a concert at UJ. The rumour is started by a group of 100 students. There are a total of 10,000 students on campus, and sooner or later, everybody hears the rumour. Assuming that the rumour spreads according to the logistic growth model, where S(t) is the number of students who have heard the rumour at time t (measured in days):

a. Write a differential equation that describes the situation above.

(1)

b. Without solving the equation, write down the solution to this equation (i.e. write down an explicit expression for S(t)) and compute the value of A.

(4)

c. After 5 days, approximately one in four students (i.e. 2500 students) have heard the rumour. Use this information to compute the value of k (to 3 d.p).

(2)

d. How long until half of the students have heard the rumour (to 1 d.p)?

(2)

e. Find $F'(0)$ to 1 d.p. and explain in words what this value is.

(2)

QUESTION 6 [8 marks]

A study in 2004 found that 23% of university students smoked cigarettes at least once a week. A public health officer wants to find out whether this percentage has decreased over the last fifteen years. She surveys 720 students across 15 universities and finds that 140 of them report smoking at least once per week.

a. State the Null and Alternative hypotheses.

(1)

b. Compute the test statistic associated with this sample (to 2 d.p).

(3)

c. Compute the p-value associated with this test statistic.

(2)

d. Can the public health officer conclude with 98% confidence that the proportion student smokers has indeed decreased? Explain why or why not.

(2)

QUESTION 7 [10 marks]

A dairy farmer has started treating his cows with a new antibiotic. Previously, average milk production was 19.3 litres per cow per day. He wants to know if using the antibiotic treatment has changed milk production in any way. He takes a sample of 65 cows over several different days, and finds a sample mean of 18.7 litres and a standard deviation of 2.3.

a) The farm wishes to know if average milk production has changed with a confidence level of 95%.

i. State the null and alternative hypotheses.

(1)

ii. Compute the relevant test statistic z_t (to 2 d.p).

(2)

iii. In the space below, sketch the rejection region for this hypothesis test, indicating the critical value(z) of z for $\alpha = 0.05$

(2)

iv. Should the farmer conclude that milk production has changed? Explain why or why not.

(2)

b) Calculate the p-value associated with the test-statistic above and conclude whether H_0 can be rejected at the 1% level of significance.

(3)

QUESTION 8 [11 marks]

Pfizer – a medical company – is doing research on a new HIV treatment that is supposed to reduce viral load. Eight people volunteer to try to the new treatment. Their viral load (in HIV per milliliter of blood) both before and after the treatment is listed below.

	Viral Load (10^3 per ml)		
Observation	**Before**	**After**	**Difference**
1	52.6	53.3	0.7
2	41.9	31.1	-10.8
3	45.4	43.3	-2.1
4	45.8	35.6	-10.2
5	39.1	13.6	-25.5
6	48	55.7	7.7
7	54.3	42.7	-11.6
8	49.4	41.9	-7.5

The researchers plan to use a paired t-test to establish if the treatment is working.

1. State the null and alternative hypotheses:

(1)

2. Use a calculator to compute:

   a. The mean of the sample differences $\bar{d}$

   (1)

   b. The standard deviation of the sample differences s (to 2 d.p).

   (1)

   c. The standard error of the sample differences SE (to 3 d.p).

   (2)

3. Compute the relevant test statistic for this experiment (to 2 d.p).

(2)

4. What critical value of t must be achieved in order to reject H_0 at the $a = 5\%$ level?

(2)

5. Should the researchers conclude that the drug appears to be working? Explain why or why not.

(2)

Standard Normal Probabilities

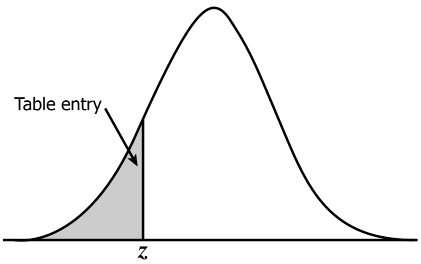


Table entry for z is the area under the standard normal curve to the left of z.

z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

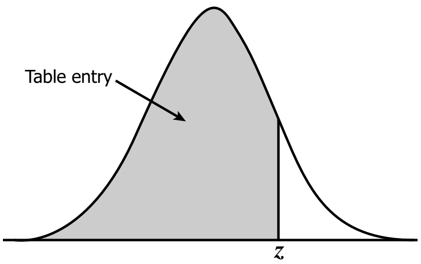
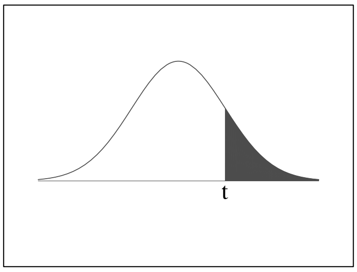


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

t-Distribution Table



The shaded area is equal to α for $t = t_{\alpha}$.

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.434	2.719
38	1.304	1.686	2.024	2.429	2.712
∞	1.282	1.645	1.960	2.326	2.576