



UNIVERSITY
OF
JOHANNESBURG

FACULTY OF SCIENCE

Department of Pure and Applied Mathematics

**Module MAT1C2E
Bio and Enviro Maths and Stats**

**Campus: APK
Assessment: Supplementary Summative Assessment**

Date:
Assessor
Internal Moderator
Duration 120 minutes
Number of pages:

Time:
Andrew Einhorn
Miles Askes
Marks 100
1-11 pages

FIRST + LAST NAME

STUDENT NUMBER

CONTACT NUMBER

INSTRUCTIONS

1. Answer all of the questions on the paper in pen. You may use pencils to sketch graphs or draw diagrams.
2. Calculators may be used on this test
3. Show all calculations and motivate all answers.
4. If you require extra space, continue your work on the adjacent blank page and indicate this clearly.
5. Unless stated otherwise, round all decimal answers to 2 significant figures.

Question 1 [5 marks]

For questions 1.1 through 1.5, mark the correct answers with an X in the box below.

	A	B	C	D	E
1.1					
1.2					
1.3					
1.4					
1.5					

1.1. An angle of 135° has a radian measure of:

- a. $\frac{3\pi}{2}$ b. $\frac{3\pi}{4}$ c. $\frac{3\pi}{7}$ d. $\frac{3\pi}{5}$ e. None of these

1.2. Which of the following functions is not increasing on its domain

- a. $f(x) = 2^x$ b. $f(x) = \tan(x)$ c. $f(x) = \log_{10} x$
d. $f(x) = x^2$ e. $f(x) = x^3$

1.3. The coefficient of the b^2 in the expansion of $(a + 2b)^3$ is:

- a. 2 b. 4 c. 6
d. 8 e. 12

1.4. Which of the following statements is true:

- a. If f is a function, then $f(s + t) = f(s) + f(t)$
b. If $f(s) = f(t)$ then $s = t$
c. If $x_1 < x_2$ and f is a decreasing function, then $f(x_1) > f(x_2)$
d. If f is a function, then $f(3x) = 3f(x)$
e. If f and g are functions, then $f \circ g = g \circ f$

1.5. Which of the following functions reflect $\ln(x)$ about the y-axis and stretch it horizontally by a factor of 2?

- a. $y = \ln\left(-\frac{x}{2}\right)$ b. $y = -\ln(2x)$ c. $y = \frac{1}{2}\ln(-x)$
d. $y = -\ln\left(\frac{x}{2}\right)$ e. $y = -2\ln(x)$

Question 2 [10 marks]

2.1 Sketch the following angles on the same set of axes below:

$$\frac{\pi}{3} \quad ; \quad \frac{7}{6}\pi \quad ; \quad -\frac{7}{5}\pi \quad ; \quad -1$$

(2)

2.2 Use your knowledge of special angles and trig graphs to compute the following:

a. $\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)$

b. $\sec\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{3}\right)$

c. $\sin\left(\frac{3}{2}\pi\right)\cos(\pi)$

(3)

2.3 Sketch the angle $\frac{4}{3}\pi$ and use your sketch to find $\cos\left(\frac{5}{3}\pi\right)$

(3)

2.4 How many radians does the minute hand of a clock move in:

a. 15 minutes

b. 10 minutes:

c. 25 minutes:

d. 50 minutes

(2)
2/11

Question 3 [10 marks]

2.1 Find the domain of the following functions:

a. $f(x) = \log_2(2x - 3)$

(1)

b. $g(x) = \frac{1}{\sqrt{x^2 - 9}}$

(2)

2.2 Use the table to evaluate each expression:

x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

a. $f(g(1))$

b. $g(f(1))$

c. $(g \circ f)(3)$

d. $(f \circ g)(6)$

(4)

2.3 Given $f(x) = \ln(\cos x)$, find $f''\left(\frac{\pi}{6}\right)$

(3)

Question 4 [10 marks]

3.1 Given the function $f(x) = x^3 - 5x$

- a. Find the $f'(x)$ using the definition of the derivative (i.e. from first principles)

(4)

- b. Find the equation of the tangent line to f when $x = 1$.

(3)

- c. Find the equation of the normal line when $x = 1$.

(1)

3.2 If an equation of the tangent line to the graph of $y = f(x)$ at the point where $x = 2$ is $y = 4x - 5$, find $f(2)$ and $f'(2)$.

(2)

Question 5 [10 marks]

Sketch the following functions, making sure to indicate any x- or y- intercepts, the scale of your axes, and any applicable asymptotes.

a. $y = \sin(2x) + 1$ on the interval $[0, 2\pi]$

(2.5)

b. $f(x) = -(x + 1)^3$

(2.5)

c. $y = -\log_2(x + 2)$

(2.5)

d. $f(x) = 2(3^{-x})$

(2.5)

Question 6 [15 marks]

5.1 Find the derivatives of the following functions:

a. $y = x^2 \sin(\pi x)$

(2)

b. $y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}}$

(2)

c. $f(\theta) = \cos(\tan(\theta))$

(2)

d. $f(x) = 3^{x \ln(x)}$

(2)

e. $y = \frac{e^{\frac{1}{x}}}{x^2}$

(2)

f. $y = \sqrt{\sin(\sqrt{x})}$

(2)

5.2 Prove that $\frac{d}{dx} \csc(x) = -\csc(x) \cot(x)$ by rewriting $\csc(x)$ in terms of basic trig function(s) and applying the quotient rule to find its derivative.

(3)

Question 7 [5 marks]

For questions 6.1 through 6.5, mark the correct answers with an X in the box below.

	A	B	C	D	E
1					
2					
3					
4					
5					

6.1 The pages of a book are labeled 1 up to 25. If I flip to a random page, what is the probability that the page number will contain a 2?

a. $\frac{1}{25}$

b. $\frac{3}{25}$

c. $\frac{5}{25}$

d. $\frac{8}{25}$

e. $\frac{9}{25}$

6.2 Thandi has 7 different pairs of shoes in her closet, but can only fit 4 of them into her bag for her holiday. How many different combinations of pairs of shoe can she pack in her bag?

a. 4

b. 28

c. 35

d. 210

e. 840

6.3 If there are 3 Red balls and 4 Green balls in a bag, and three balls are drawn consecutively without replacement, what is the probability that all three will be red?

a. $\frac{1}{35}$

b. $\frac{1}{8}$

c. $\frac{3}{7}$

d. $\frac{1}{7}$

e. $\frac{81}{243}$

6.4 If the probability of A and B occurring is $\frac{1}{6}$, the probability of A or B occurring is $\frac{2}{3}$, and $P(B) = \frac{1}{4}$, then $P(A)$ is equal to:

a. $\frac{1}{2}$

b. $\frac{7}{12}$

c. $\frac{1}{6}$

d. $\frac{3}{4}$

e. $\frac{3}{8}$

6.5 If $p(A \cap B) = 0$, then which of the following statements are true:

- i. A and B are independent
- ii. A and B are mutually exclusive
- iii. $p(A \cup B) = p(A) + p(B)$

- a. (i) only
- b. (i) and (ii)
- c. (ii) and (iii)
- d. (i) and (iii)
- e. (i), (ii) and (iii)

Question 8 [10 marks]

- 7.1. The peak temperatures measured in Cape Town for the 30 days in December are tabulated below ($^{\circ}\text{C}$) in ascending order.

21.3	22.2	23.5	24.1	24.7	25.1
25.6	26.2	26.7	26.9	27.4	27.5
28.2	28.7	29.6	29.9	30.5	30.7
31.2	31.5	31.5	31.9	32.8	33.0
34.5	34.6	35.7	36.2	36.7	38.3

- a. Use this data to complete the table below. Begin your first interval at 20, use a class width of 3, and use the rule of left inclusion for your intervals.

Interval	Frequency	Relative Freq (2 d.p)

(3)

- b. Use this data to construct a relative frequency histogram below. Label your axes and give your chart an appropriate title.

(3)

- c. If this data is representative of a normal December, what percentage of days have a peak temperature of more than 29 degrees?

(1)

- 7.2 In the space below, manually calculate the standard deviation for the following five numbers: 13, 17, 19, 22, 24. Show all working (i.e. it is not sufficient to simply use a calculator to find the s.d.)

(3)

Question 9 [10 marks]

Mr Machedi tabulates the test scores (out of 100) for the 25 students in his class:

27	48	52	54	57
58	60	62	62	65
67	68	69	71	72
74	74	75	78	80
83	84	87	91	92

- a. Find Q1, Q2, Q3 and the interquartile range.

(4)

- b. Compute the lower and upper bound for outliers.

(2)

- c. Construct a box and whisker plot to display this information. Use a scale of 0 – 100, label your axes and indicate outliers (if present).

(3)

- d. Calculate the percentile of the student who scored 78.

(1)

Question 10 [10 marks]

10.1 Consider a regular deck of playing cards with 4 suits, hearts (H), diamonds (D), clubs (C) and spades (S). Suppose we pick one card, observe its suit, and replace it. We Then pick a sick card, observe its suit, and replace it. Define:

- A: Get at least one heart
 B: Do not get any spades

a. Use a table to enumerate all of the equally likely outcomes (e.g. HH, HS, HD, HC, etc)

(1)

b. Calculate the following probabilities (6 marks):

$p(A)$	$p(B)$
$p(A \cap B)$	$p(A B)$
$p(A \cup B)$	$p(B A^c)$

10.2 A bag contains 6 black balls and 5 white balls. If I pick 3 balls without replacement, calculate the probabilities that:

a. All three balls are black.

(1)

b. Two are black and one is white.

(2)