



UNIVERSITY
OF
JOHANNESBURG

FACULTY OF SCIENCE

Department of Pure and Applied Mathematics

**Module MAT1C2E
Bio and Enviro Maths and Stats**

**Campus: APK
Assessment: Final Summative Assessment**

Date: 22 November 2019
Assessor
Internal Moderator
Duration 120 minutes
Number of pages: 1-12 pages

Time: 14h00
Andrew Einhorn
Miles Askes
Marks 95

FIRST + LAST NAME

STUDENT NUMBER

CONTACT NUMBER

TUTORIAL GROUP

INSTRUCTIONS

1. Answer all of the questions on the paper in pen. You may use pencils to sketch graphs or draw diagrams.
2. Calculators may be used for this test.
3. Show all calculations and motivate all answers.
4. If you require extra space, continue your work on the adjacent blank page and indicate this clearly.
5. Unless stated otherwise, round all decimal answers to 2 significant figures.

Question 1 [5 marks]

For questions 1.1 through 1.5, mark the correct answers with an X in the box below.

	A	B	C	D	E
1.1					
1.2					
1.3					
1.4					
1.5					

1.1. An angle of 105° has a radian measure of:

- a. $\frac{8\pi}{3}$ b. $\frac{7\pi}{12}$ c. $\frac{12\pi}{7}$ d. $\frac{6\pi}{11}$ e. None of these

1.2. Which of the following is the result of the graph of $f(x) = e^x$ being stretched horizontally by a factor of 2 and reflected about the y-axis?

- a. $f(x) = e^{\frac{1}{2}x}$ b. $f(x) = -2e^x$ c. $f(x) = 2e^{-x}$
d. $f(x) = e^{-\frac{x}{2}}$ e. None of these

1.3. Over the course of five hours, the hour hand of the clock moves:

- a. $\frac{\pi}{5} \text{ rad}$ b. $\frac{2}{5}\pi \text{ rad}$ c. $\frac{5}{12}\pi \text{ rad}$
d. $\frac{5}{6}\pi \text{ rad}$ e. $5\pi \text{ rad}$

1.4. The tangent line to the graph of a function f at the point $(3, 4)$ passes through the point $(7, 8)$. Find $f'(3)$

- a. -1 b. 0 c. 1
d. 2 e. $\frac{3}{5}$

1.5. The coefficient of the a^2 term in the expansion of $(2a + 3)^3$ is:

- a. 8 b. 12 c. 24 d. 36 e. 48

Question 2 [10 marks]

2.1 Sketch the following angles on the same set of axes below:

$$\frac{5}{8}\pi ; -\frac{3}{4}\pi ; 1 ; \frac{11}{6}\pi$$

(2)

2.2 Use your knowledge of special angles and trig graphs to compute the following:

a. $\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right)$

b. $\sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)$

c. $\sin\left(\frac{3}{2}\pi\right)\cos(\pi)$

(3)

2.3 Sketch the angle $\frac{5}{3}\pi$ and use your sketch to find $\sin\left(\frac{5}{3}\pi\right)$

(3)

2.4 How many radians does the minute hand of a clock move in:

a. Half an hour:

b. 20 minutes:

c. 75 minutes:

d. Seven and a half hours :

(2)

Question 3 [10 marks]

3.1 Find the domain of the following functions:

a. $f(x) = \frac{1}{x^2 - 9x + 14}$

b. $g(x) = \ln(25 - x^2)$

(1)

(2)

3.2 Given $f(x) = x^2$ and $g(x) = \sqrt{3 - x}$

a. Find $(f \circ g)(x)$ and its domain

(2)

b. Find $(g \circ f)(x)$ and its domain

(2)

3.3 Given $f(t) = 2\sin^2 t$, find $f''\left(\frac{\pi}{6}\right)$

(3)

Question 4 [10 marks]

4.1 Given the function $f(x) = \sqrt{4 + 3x}$

- a. Find the $f'(x)$ using the definition of the derivative (i.e. from first principles)

(3)

- b. Find the domain of f and of f' .

(2)

- c. Find the equation of the tangent line to f at the point $(4,4)$.

(2)

- d. Find the equation of the normal line to f at the point $(4,4)$.

(1)

4.2 Find an equation of the tangent line to the graph $y = g(x)$ at $x = 5$ if $g(5) = -3$ and $g'(5) = 4$.

(2)

Question 6 [15 marks]

6.1 Find the derivatives of the following functions:

a. $y = x^2 \cos(x)$

(2)

b. $y = \frac{\ln(x)}{x^3 + 2x}$

(2)

c. $y = \ln(x \ln x)$

(2)

d. $y = \tan\left(\frac{t}{1+t^2}\right)$

(2)

e. $y = \sqrt{x} \sin \sqrt{x}$

(2)

f. $y = 2^{x \sec x}$

(2)

6.2 Prove that $\frac{d}{dx} \cot(x) = -\csc^2(x)$ by rewriting $\cot(x)$ in terms of $\sin(x)$ and $\cos(x)$ and applying the quotient rule to find its derivative.

(3)

Question 7 [5 marks]

For questions 1 through 5, mark the correct answers with an X in the box below.

	A	B	C	D	E
6.1					
6.2					
6.3					
6.4					
6.5					

7.1. If two dice are rolled, the probability of seeing at least one five is:

- a. $\frac{1}{36}$ b. $\frac{5}{36}$ c. $\frac{6}{36}$ d. $\frac{11}{36}$ e. $\frac{12}{36}$

7.2. In how many ways can a committee of three people be selected from a pool of seven candidates?

- a. 6 b. 21 c. 35
d. 210 e. 840

7.3. Suppose you are asked to list, in order of preference, the three best movies you have seen this year. If you saw 10 movies during the year, in how many ways can the three best be chosen and ranked?

- a. $3!$ b. $10!$ c. $\frac{10!}{7!}$
d. $\frac{10!}{3!7!}$ e. None of these

7.4. If $p(A) = \frac{1}{3}$, $p(B) = \frac{1}{2}$ and we are told that $p(A \cap B) = \frac{1}{4}$, then $p(A \cup B) =$

- a. $\frac{7}{12}$ b. $\frac{1}{3}$ c. $\frac{5}{12}$
d. $\frac{5}{6}$ e. 1

7.5. If $p(A \cap B) = 0$, then which of the following statements are true:

- i. A and B are independent
 - ii. A and B are mutually exclusive
 - iii. $p(A \cup B) = p(A) + p(B)$
- a. (i) only
b. (i) and (ii)
c. (ii) and (iii)
d. (i) and (iii)
e. (i), (ii) and (iii)

Question 8 [10 marks]

A clinic doing research on recovery time from influenza finds the following recovery times (in days) for 22 of their patients.

3	5	7	7	7	8
8	9	9	9	9	9
9	10	10	10	10	12
12	13	16	19		

a. Find the mean (to 2d.p), median, mode and range of this dataset:

Mean =

Median =

Mode =

Range = (2)

b. Compute the three quartiles Q1, Q2 and Q3, as well as the interquartile range.

(3)

c. Compute the upper and lower boundaries for outliers in this data set.

(2)

d. Construct a box and whisker plot to display this dataset (label axes, scale and outliers if present).

(3)

Question 9 [8 marks]

Sameera asks each of the 24 students in her class how many cousins they have. Their responses are tabulated below

0	2	3	4	4	5
6	7	8	9	9	10
10	11	11	11	12	12
12	13	14	15	16	18

- 9.1 In the space below, create a table with columns for class interval, frequency and relative frequency. Use a class width of 4, begin your first interval at 0, and use the method of left inclusion.

(3)

- 9.2 Using your table, create a relative frequency histogram for this data. Give a label and scale to both axes and give the histogram a title.

(3)

- 9.3. What percentage of students have between 10 and 15 cousins?

(1)

- 9.4 Assuming this is a fair representation of family size, what is the probability that a randomly selected student at the school will have fewer than 5 cousins?

(1)

Question 10 [12 marks]

10.1 A five-sided dice shows either 1, 2, 3, 4 or 5 with equal probability. Suppose we roll two five-sided dice and define the following events:

- A: Observe at least one 5
- B: Observe neither a 1 nor a 2

Calculate the following probabilities:

- a. $p(A)$
- b. $p(B)$
- c. $p(A \cap B)$
- d. $p(A \cup B)$
- e. $p(A|B)$
- f. $p(B|A^c)$

(7)

10.2 I have ten smarties in a box: 3 Red, 3 Green and 4 Blue. If I take three smarties out at random, what is the probability that:

- a. All three are blue?
- b. I get two blues and one red?
- c. I get one of each colour?

(3)

10.3 If I draw two cards from a regular deck of playing cards, what is the probability that:

- a. Both show the same value (e.g. (5, 5) or (Q, Q))
- b. Both are of the same suit.

(2)