UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE	MAT01B1/MAT1B01 APPLICATIONS OF CALCULUS
CAMPUS	APK
ASSESSMENT	SUPPLEMENTARY EXAMINATION

DATE	JANUARY 2020
ASSESSOR(S)	DR A SWARTZ DR C ROBINSON
INTERNAL MODERATOR	DR A CRAIG
DURATION 120 MINUTES	MARKS 70

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 14 PAGES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [5 marks]

For questions 1.1 – 1.5, choose the correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					

1.1 Determine whether the following statement is true or false: (1)

The partial fraction decomposition of $\dfrac{x^2+4}{x^2(x-4)}$ is $\dfrac{A}{x^2}+\dfrac{B}{x-4}$.

(a) True

(b) False

1.2 A function f has a local maximum or minimum at a number c. Which of the following statements is false? (1)

(a) $f'(c)$ always exists and equals zero.

(b) The derivative of f takes the value 0 at c, if it exists.

(c) The number c lies in the domain of f.

(d) The derivative of f may not exist at c.

(e) None of the above.

1.3 Find the number a such that the average value of the function $f(x)=\dfrac{1}{x}$ on the interval $[1,a]$ is equal to $\dfrac{1}{e-1}$. (1)

(a) $\dfrac{1}{e-1}$

(b) 1

(c) e

(d) $\ln(e-1)$

(e) None of the above.

1.4 Which of the following is *not* a solid of revolution? (1)

(a) Sphere.

(b) Pyramid.

(c) Cone.

(d) Cylinder.

(e) None of the above.

1.5 Let $G(x) = \dfrac{d}{dx} f(x)$. What quantity is described by the integral $\displaystyle\int_a^b \sqrt{1 + [G'(x)]^2}\, dx$? (1)

(a) The arc length of the curve $G(x)$ from $x = a$ to $x = b$.

(b) The arc length of the curve $f(x)$ from $x = a$ to $x = b$.

(c) The arc length of the curve $\sqrt{1 + G(x)}$ from $x = a$ to $x = b$.

(d) The arc length of the curve $[f(x)]^2$ from $x = a$ to $x = b$.

(e) None of the above.

Question 2 [4 marks]

For questions 2.1 – 2.2, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
2.1					
2.2					

2.1 If f is continuous, then $\displaystyle\int_{-\infty}^{\infty} f(x)\ dx = \lim_{t\to\infty} \int_{-t}^{t} f(x)\ dx$. (2)

(a) True

(b) False

2.2 The inflection point of the function $f(x) = 2xe^{-3x}$ is at which x value? (2)

(a) $\frac{1}{3}$

(b) 2

(c) $\frac{2}{3}$

(d) -3

(e) None of the above statements are correct.

Question 3 [14 marks]

Evaluate the following integrals:

(a) $\int e^{\cos t} \sin 2t \ dt$ (4)

(b) $\int \cot^4 \theta \csc^4 \theta \ d\theta$ (3)

(c) $\int \frac{\sqrt{1+x^2}}{x}\, dx$ (4)

(d) $\int \frac{1}{(t+4)(t-1)}\, dt$ (3)

Question 4 [4 marks]

(i) Sketch the region R bounded by the curve $x = y^2 - 2y - 3$ and $x = 0$. (2)

(ii) Find the area of the region R. (2)

Question 5 [3 marks]

Consider the integral $I = \int_0^1 \frac{1}{x}\,dx$. Determine whether the integral is convergent or divergent.

Question 6 [2 marks]

A spherical balloon is being deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate is air escaping when the radius is 9 cm? HINT: The volume of a sphere or radius r is given by $V = \frac{4}{3}\pi r^3$.

Question 7 [3 marks]

Sketch the graph of a function f which satisfies the following conditions.

(a) The function is even.

(b) $\lim_{x\to 2^+} f(x) = \infty$ and $\lim_{x\to 2^-} f(x) = -\infty$

(c) $f(0) = 0$

(d) $f'(x) < 0$ for $x \in (0, 2)$

(e) $\lim_{x\to\infty} f(x) = 1$

Question 8 [2 marks]

Find two numbers whose difference is 170 and whose product is a minimum.

Question 9 [4 marks]

Consider the region R bounded by $y = \sqrt{1 - x^2}$ and $x + y = 1$.

(a) Sketch the region R, indicating the points of intersection. (1)

(b) Use the **Cylindrical Shells Method** to find the volume of the solid obtained by rotating the region R about the x-axis. (3)

Question 10 [3 marks]

Find the area of the surface of revolution obtained by rotating the curve $x = \frac{y^4}{8} + \frac{1}{4y^2}$ about the y-axis between the two points $\left(\frac{3}{8}, 0\right)$ and $\left(\frac{33}{16}, 1\right)$. Simplify the integrand as far as possible. **DO NOT INTEGRATE.**

Question 11 [6 marks]

(a) Solve the differential equation $\left(\csc x + \frac{2}{\sin x}\right)\frac{dy}{dx} = 8$. Write your solution in the form $y = f(x)$. (2)

(b) Solve the initial value problem: (4)

$$x^2y' - xy = x^2 + 4, \qquad x > 0, \qquad y(1) = 0$$

Question 12 [6 marks]

(a) Consider the parametric equations below:

$$x = e^{\sqrt{t}}, \qquad y = t - \ln t^6$$

Find an equation of the tangent to the curve at the point $t = 1$. (3)

(b) Sketch the graph of the polar equation (3)

$$r = 2\sin 3\theta.$$

Question 13 [4 marks]

Consider the matrices below and then write down the following:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

(a) $A + B$ (1)

(b) B^T (1)

(c) BA (2)

Question 14 [3 marks]

Consider the system of equations below:

$$\begin{aligned} 2x - 3y + 4z &= 1 \\ -x + 2y &= 3 \\ y + 3z &= 4 \end{aligned}$$

(a) Represent the system of equations as an augmented matrix. (0.5)

(b) Use elementary row operations to write the augmented matrix in reduced row echelon form and write down the solutions to the equations. (2.5)

Question 15 [3 marks]

Use the Binomial Theorem to expand $\left(2a - \frac{1}{2a}\right)^5$. Simplify all coefficients. (3)

Question 16 [4 marks]

(a) State Fermat's Theorem. (2)

(b) State the Mean Value Theorem for Integrals. (2)