UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE: MAT2B10/MAT01B2
MULTIVARIABLE AND VECTOR CALCULUS

CAMPUS: APK
ASSESSMENT: SUPPLEMENTARY EXAMINATION

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ASSESSOR(S): DR C. RATHILAL
MR M. SIAS

INTERNAL MODERATOR: DR A. GOSWAMI

DURATION: 120 MINUTES **MAXIMUM MARKS: 50**

SURNAME AND INITIALS ____________________

STUDENT NUMBER ____________________

CONTACT NUMBER ____________________

NUMBER OF PAGES: 1 + 11 PAGES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [4]

Choose the correct option for the multiple choice questions below and **write your answer in the table provided**.

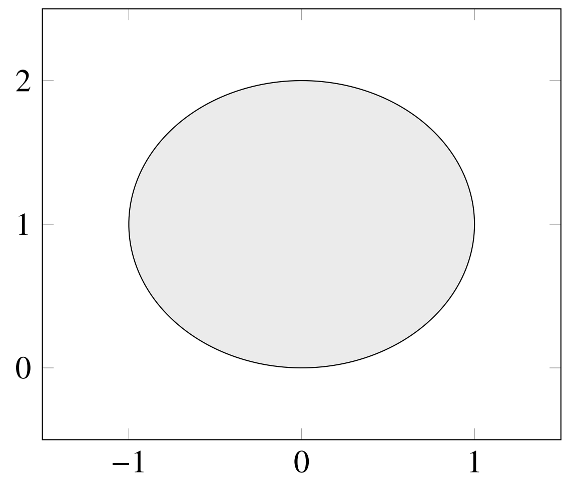
Question	Answer
1.1	
1.2	
1.3	
1.4	

1.1. Name the quadric surface represented by the equation (1)

$$-x^2 + 6z + 4x + y^z + z^2 = -5.$$

(a) Ellipsoid (b) Cone (c) Elliptic paraboloid (d) Cylinder

1.2. An iterated integral which represents *half* of the area of the region below is given by: (1)



(a) $\int_0^{2\pi}\int_0^{2\sin\theta} r\,drd\theta$ (b) $\int_{\pi/4}^{3\pi/4}\int_0^{2\sin\theta} r\,drd\theta$ (c) $\int_0^{\pi/2}\int_0^{2\cos\theta} r\,drd\theta$ (d) $\int_{\pi/2}^{\pi}\int_0^{2\sin\theta} r\,drd\theta$

1.3. Let f be a scalar field and let $\mathbf{F}$ be a vector field. Which of the expressions below are meaningful *and* results in a vector field: (1)

(i) $(\operatorname{div}\mathbf{F})\,\nabla f$
(ii) $\nabla f \times \operatorname{div}\mathbf{F}$
(iii) $\operatorname{div}(\operatorname{curl}(\nabla f))$
(iv) $\operatorname{curl}(\operatorname{curl}\mathbf{F})$
(v) $\operatorname{curl}\mathbf{F}\cdot\operatorname{curl}\mathbf{F}$

(a) i, iv (b) iii, v (c) iv (d) iv, v (e) None of these choices

1.4. Evaluate the line integral given by $\oint_C y^3\,dx - x^3\,dy$, where C is the circle $x^2 + y^2 = 4$. (1)

(a) -12π (b) -24π (c) 24π (d) 18π (e) -18π

Question 2 [3]

Use the Squeeze Theorem to find the limit below:

$$\lim_{(x,y)\to(0,0)} \frac{x^6 \sin^2\left(y + \frac{\pi}{2}\right)}{x^4 + y^4}.$$

Question 3 [5]

Suppose that f is a differentiable function of x and y. Prove that f has a directional derivative in the direction of any unit vector $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ and

$$\mathrm{D}_{\mathbf{u}} f(x, y) = f_x(x, y)a + f_y(x, y)b.$$

Question 4 [6]

In evaluating a double integral over a region D, a sum of iterated integrals was obtained as follows:

$$\int_{-2}^{-\sqrt{2}} \int_{-\sqrt{4-y^2}}^{0} \arctan\left(\frac{y}{x}\right) dx\, dy + \int_{-\sqrt{2}}^{-1} \int_{y}^{0} \arctan\left(\frac{y}{x}\right) dx\, dy + \int_{-1}^{-1/\sqrt{2}} \int_{y}^{-\sqrt{1-y^2}} \arctan\left(\frac{y}{x}\right) dx\, dy.$$

Sketch the region D and then evaluate the double integral by first converting it to polar coordinates.

Question 5 [11]

(5.1) Evaluate the following integral by changing to cylindrical coordinates:

$$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\, dzdxdy$$

(6)

(5.2) Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy−plane, and below the cone $z = \sqrt{x^2 + y^2}$. (5)

Question 6 [4]

Use the change of variables $x = u^2 - v^2$, $y = 2uv$ to evaluate the integral $\iint_R y\, dA$, where R is the region bounded by the $x-$ axis and the parabolas $y^2 = 4 - 4x$ and $y^2 = 4 + 4x$, $y \geq 0$.

Hint: The region R is the image of the square S, where $S = [0, 1] \times [0, 1]$.

Question 7 [4]

Let $D \subseteq \mathbb{R}^2$ be the region $1 \leq x^2 + y^2 \leq 4, x \geq 0, y \geq 0$, and let C be the boundary curve of D oriented clockwise. Using Green's Theorem, evaluate the integral

$$\oint_C \sqrt{x^2 + y^2}dx + \ln\left(x + \sqrt{x^2 + y^2}\right)dy.$$

Question 8 [6]

Consider the vector field $\mathbf{F} : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$, defined by $\mathbf{F}(x, y, z) = (3x^2, 2xz - y, z), \quad (x, y, z) \in \mathbb{R}^3$.

(8.1) Evaluate the following line integrals in the direction of increasing values of "t". (5)

(i) $\int_\phi \mathbf{F}(x, y, z)d\mathbf{r}$, where $\phi(t) = (2t^3, t, t^3)$, $t \in [0, 1]$.

(ii) $\int_\psi \mathbf{F}(x, y, z)d\mathbf{r}$, where $\psi(t) = (2t, t^3, t^2)$, $t \in [0, 1]$.

(8.2) Is $\mathbf{F}(x, y, z)$ a gradient field? Justify your answer using (8.1). (1)

Question 9 [7]

Consider the vector field $\mathbf{F} : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$, defined by

$$\mathbf{F}(x, y, z) = (z^2 - e^y \sin x, e^y \cos x + 2y, 2xz), \ (x, y, z) \in \mathbb{R}^3.$$

(9.1) Show that $\mathbf{F}(x, y, z)$ is a conservative vector field, and find a potential function for $\mathbf{F}(x, y, z)$. (5)

(9.2) Evaluate $\int_C (z^2 - e^y \sin x)dx + (e^y \cos x + 2y)dy + (2xz)dz$, where C is the smooth curve from $(0, 1, -1)$ to $(\pi, 0, -2)$. (2)