UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: MAFT0B3/MA3BFET

COURSE: MATHEMATICS 3B FOR TEACHERS

CAMPUS: APK

EXAM: MAIN EXAMINATION - NOVEMBER 2019

DATE: MONDAY 11 NOVEMBER 2019

TIME: 12:30 – 15:30

ASSESSOR: MR. A. ALOCHUKWU

EXTERNAL MODERATOR: PROF. O. IGHEDO

DURATION: 3 HOURS **MARKS:** 130

SURNAME AND INITIALS

STUDENT NUMBER

CONTACT NUMBER

NUMBER OF PAGES: 18 PAGES (including front page)

INSTRUCTIONS: ANSWER ALL THE QUESTIONS, SHOW ALL CALCULATIONS, CALCULATORS ARE NOT ALLOWED.

Section A: Matrices and Systems of Linear Equations [43]

Question 1:

Give answers to the following questions. (7)

Statement	**Answer (& Explanation)**
Given a system of linear equations, the matrix derived from the system is called the (a)---------- matrix while the matrix derived from the coefficients of the system of linear equations is called the (b) ----------- matrix of the system.	a) b)
The $n \times n$ matrix consisting of 1's on its main diagonal and 0's elsewhere is called the (a)-------- matrix of order (b)-------------.	a) b)
The method of using determinants to solve a system of linear equations is popularly known as tbe Gauss Jordan Elimination Method (True or False)	
If there exists an $n \times n$ matrix A^{-1} such that $AA^{-1} = I_n = A^{-1}A$, then A is said to be (a) ------ or (b)------- and A^{-1} is called the (c)---------- of A.	a) b) c)
Determine whether the matrix is in row-echelon form. If it is, determine whether it is in reduced row-echelon form. $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & -\mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{2} \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{2} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix}$	
If B is an invertible matrix, then the system of linear equations represented by $BY = C$ has a unique solution given by $Y = CB^{-1}$ (True or False: justify your choice of answer)	

Question 2:

2.1 An electronics store in Harare sells two models of laptop computers. Because of the demand, the store stocks at least twice as many units of model A as of model B. The costs to the store for the two models are $\$800$ and $\$1200$, respectively. The management does not want more than $\$20,000$ in computer inventory at any one time, and it wants at least four model A laptop computers and two model B laptop computers in inventory at all times. Find and graph a system of inequalities describing all possible inventory levels. (6)

2.2 Given the system of inequalities:

$$\begin{cases} \boldsymbol{x \geq 0} \quad , \quad \boldsymbol{y \geq 0} \\ \boldsymbol{x \leq 10} \quad , \quad \boldsymbol{y \leq 20} \\ \boldsymbol{x + y \quad \geq \quad 5} \\ \boldsymbol{x + 2y \quad \leq \quad 18} \end{cases}$$

2.2.1 Draw a graphical representation of the system and find the feasible region.

(5)

2.2.2 Use the objective function $\boldsymbol{Q = 70x + 80y}$ and the graphical representation (in question 2.2.1) to find the maximum value for $\boldsymbol{Q}$.

(4)

Question 3:

3.1 Net Florist located in Sandton is creating 10 centrepieces for the table at a wedding reception. Roses cost $\$2.50$ each, lilies cost $\$4$ each, and irises cost $\$2$ each. The customer has a budget of $\$300$ allocated for the centrepieces and wants each centrepiece to contain 12 flowers, with twice as many roses as the number of irises and lilies combined.

a) Write a system of linear equations that represents the situation.
b) Write a matrix equation that corresponds to your system.
c) Solve your system of linear equations using an inverse matrix. Find the number of flowers of each type that the florist can use to create the 10 centrepieces. (8)

3.2 Given:

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \boldsymbol{z} & \boldsymbol{z}^2 \\ \mathbf{1} & \boldsymbol{y} & \boldsymbol{y}^2 \\ \mathbf{1} & \boldsymbol{x} & \boldsymbol{x}^2 \end{bmatrix}$$

3.2.1 Find Cofactor C_{23} (3)

3.2.2 Evaluate the determinant, to verify $\boldsymbol{det(A)} = (\boldsymbol{y} - \boldsymbol{x})(\boldsymbol{z} - \boldsymbol{x})(\boldsymbol{y} - \boldsymbol{z})$**,** by expanding along the **third row** (show all calculations).

(5)

3.2.3 State **Cramer's Rule**:

(2)

3.2.4 Hence, solve for $\boldsymbol{y}$ in the system by using **Cramer's Rule**: (3)

$$\begin{cases} \mathbf{4}\boldsymbol{x} - \mathbf{3}\boldsymbol{y} & = & -\mathbf{10} \\ \mathbf{6}\boldsymbol{x} + \mathbf{9}\boldsymbol{y} & = & \mathbf{12} \end{cases}$$

Section B: Complex Numbers **[20]**

Question 4:

4.1.1 State **DeMoivre's Theorem** for powers of complex numbers:

(2)

4.1.2 Hence, calculate and simplify the answer.

$$\left[2\left(\cos\frac{4\pi}{15}+i\sin\frac{4\pi}{15}\right)\right]^5$$

(5)

4.2.1 State the definition for **finding nth Roots of a Complex Number**:

(2)

4.2.2 Hence, solve the equation and represent the solutions graphically:

$$\boldsymbol{x^3 - (1 - i) = 0}$$

(5)

4.3 Convert the point $\left(-\mathbf{4}, -\frac{\boldsymbol{\pi}}{\mathbf{4}}\right)$ to rectangular coordinates.

(2)

4.4 Find the 1444^{th} term of the sequence where $i^2 = -1$, given that

$$\boldsymbol{b_n} = \frac{(-\mathbf{1})^{\boldsymbol{n+2}}(\mathbf{1} - \mathbf{12}\boldsymbol{i}^{\mathbf{3}\boldsymbol{n}+\mathbf{1}})}{\boldsymbol{i}^{\boldsymbol{n}+\mathbf{2}}}$$

(4)

Section C: Conic Sections & Vectors **[30]**

Question 5:

5.1. Complete the sentence:

A ______________________ is the set of all points in the plane, the difference of whose distances from two fixed points (the foci) is a constant.

(1)

5.2 Find the equation of the parabola that has its vertex at the origin and directrix given as $x = -8$. Draw a graph of the parabola.

(4)

5.3 Given the equation of a conic section:

$$\frac{1}{2}x^2 + \frac{1}{8}y^2 = \frac{1}{4}$$

5.3.1 Identify the conic section **and** write the equation in standard form.

(2)

5.3.2 Sketch a graph to represent the conic section and indicate all key points on its graph (show all necessary calculations).

(6)

5.4 Hyperbolas are called **confocal** if they have the same foci. Show that the hyperbolas are confocal for $k = 2$ and $k = 4$:

$$\boldsymbol{\frac{y^2}{k} - \frac{x^2}{16 - k} = 1}$$

(5)

5.5 Find the equation of the asymptotes for the conic section

$$\frac{\boldsymbol{x^2}}{\mathbf{16}} - \frac{\boldsymbol{y^2}}{\mathbf{9}} = \mathbf{1}$$

(2)

Question 6:

6.1 Given vector $\overline{\boldsymbol{u}} = \langle -4,2 \rangle$ and vector $\overline{\boldsymbol{v}} = \langle 5,1 \rangle$; find $-\|\overline{\boldsymbol{u}} + \overline{\boldsymbol{v}}\|$.

(3)

6.2 Find the direction (in degrees) of the vector $\overline{\boldsymbol{w}} = \langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle$.

(2)

6.3 A jet has an airspeed of **724 km/h** at a bearing of $\mathbf{135^\circ}$ $(S45^\circ E)$. The wind velocity is **10 km/h** in the direction $\boldsymbol{N30^\circ E}$. Find the resultant speed of the jet (in the wind). Draw a diagram to represent the solution.

(5)

Section D: Logic & Sets **[37]**

Question 7:

7.1 If A is true, B is false and C is true, determine the truth-value of the compound proposition (do not use a truth table):

$$\big((\boldsymbol{A} \to \boldsymbol{B}) \vee \boldsymbol{C}\big) \leftrightarrow (\neg \boldsymbol{C} \longrightarrow \boldsymbol{B})$$

(3)

7.2 Assume x is a particular real number, then use De Morgan's laws to write a negation for the statement:

$$-2 < x < 7$$

(1)

7.3 Let $\boldsymbol{S} = \{\boldsymbol{-1}, \boldsymbol{0}, \boldsymbol{1}, \dots, \boldsymbol{11}\}$, determine the truth-value of each of the following statements:

7.3.1 $\exists x \in S(x^2 - 2 < 0)$

(1)

7.3.2 $\forall x \in S(x + 1 > 0)$

(1)

Question 8:

Determine whether the argument below is valid or invalid by using a truth table (follow the guidelines provided): *If I get a Christmas bonus, I'll buy a stereo*

If I sell my motorcycle, I'll buy a stereo

∴ *If I get a Christmas bonus or I sell my motorcycle, then I'll buy a stereo*

8.1 Symbolic form of argument: (1)

8.1.1. Truth Table: (4)

				Premises		**Conclusion**

8.1.2 Truth value of argument: (1)

8.2 Use the sentence below and indicate the required forms (indicate the rule in the left column and rewrite the sentence in the correct form in the right column): (8)

Catching the 6:50 bus is sufficient for my being on time for class.

Rule	**Correct form of sentence**
Inverse	
Contrapositive	
Converse	
Negation	

Question 9:

9.1 Draw a Venn diagram for the universal set U with sets A, B and C that satisfy the given condition:

$$A \subseteq B; C \subseteq B; A \cap C = \emptyset.$$

(3)

9.2 Let $A = \{x, y, z, w\}$ and $B = \{a, b\}$. List the elements of $A \times B$.

(2)

9.3 The following is a sketch of a formal proof for sets A and B, such that:

$$\boldsymbol{A - B \subseteq A}$$

Proof: Suppose A and B are any sets and $x \in A - B$. [*We must show that* **(1)**.] By definition of set difference, $x \in$ **(2)** and $x \notin$ **(3)** . In particular, $x \in$ **(4)** [*which is what was to be shown*].

Fill in the blanks for numbers 1 to 4 in the table provided.

Numbers	**Answer**
1	
2	
3	
4	

(4)

9.4 Let the universal set be $\mathbb{R}$ of all real numbers and let $\boldsymbol{A} = \{\boldsymbol{x} \in \mathbb{R} | \boldsymbol{-3 \leq x \leq 0}\}$, $\boldsymbol{B} = \{\boldsymbol{x} \in \mathbb{R} | \boldsymbol{-1 < x < 2}\}$ and $\boldsymbol{C} = \{\boldsymbol{x} \in \mathbb{R} | \boldsymbol{6 < x \leq 8}\}$. Find each of the following:

9.4.1 $A \cup B$

(1)

9.4.2 $A \cap C$

(1)

9.4.3 A^c

(1)

9.4.4 B^c

(1)

9.4.5 $A^c \cap B^c$

(1)

9.5 Indicate which of the following relationships are **true** and which are **false**: (3)

Statement	True or False
$\mathbb{Z}^+ \subseteq \mathbb{Q}$	
$\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$	
$\{0,1,2\} = \{x \in \mathbb{R} \| -1 < x < 3\}$	
$\{1\} \subseteq \{1,2\}$	
$1 \in \{\{1\}, 2\}$	
$\{1\} \nsubseteq \{1\}$	