UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE	**MAFT0B2/MA2BFET** **MATHEMATICS 2B FOR TEACHERS**
CAMPUS	**APK**
ASSESSMENT	**SUPPLEMENTARY EXAM**

DATE JANUARY 2020	**TIME TBA**
ASSESSOR(S)	**MR T. MUDZIIRI SHUMBA**
MODERATOR	**MS S. RICHARDSON**
DURATION 120 MINUTES	**MARKS 80**

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

CONTACT NUMBER ______________________________

NUMBER OF PAGES: 1 + 11 PAGES

INSTRUCTIONS:

1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. CALCULATORS ARE **NOT ALLOWED**.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE FACING BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [7 marks]

For questions 1.1 – 1.4, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**A**	**B**	**C**	**D**	**E**
1.1					
1.2					
1.3					
1.4					

1.1 Let f be a function defined on an interval $D = [a, b]$, and let $I \subset D$ be a sub-interval. Suppose that for all $x \in I, f(x) \leq f(c_0)$ where $c_0 \in I$ and for some $c \in (a, b), f(x) \leq f(c)$ for all $x \in D$. Which of the following statements is not correct? $\boxed{1}$

(A) $f(c_0)$ is a local maximum of f

(B) $f(c_0)$ is an absolute maximum of f

(C) $f(c)$ is an absolute maximum of f

(D) $f(c)$ is a local maximum of f

(E) None of the above are incorrect.

1.2 The inflection point of the function $f(x) = xe^{-3x}$ is at $x =$? $\boxed{2}$

(A) $\frac{1}{3}$

(B) 3

(C) $\frac{2}{3}$

(D) 0

(E) None of the above.

1.3 Let f be the function defined by $f(x) = \frac{1}{2}\ln|x^2 - 1|$. Which of the following statements is incorrect? $\boxed{2}$

(a) $x = 0$ is a critical number of f.

(b) $x = 1$ is a critical number of f.

(c) $x = -1$ is not a critical number of f.

(d) $x = 0$ is a turning point of f.

(e) None of the above.

1.4 Given that $f(0) = -1,\ f'(0) = -2,\ g(0) = -3$ and $g'(0) = 4$, the slope of the tangent line to the curve $y = \frac{f(x)}{(g(x)+2)}$ at $x = 0$ is:

(A) -2.

(B) 1.

(C) 6.

(D) -6.

(E) None of the above.

Question 2 [7 marks]

(a) State the domain of the function

$$h(t) = \frac{2}{3t-3}.$$

$\boxed{1}$

(b) Give the domain of the function $g(x) = \frac{1}{\sqrt{x^2-25}}$. $\boxed{2}$

(c) State whether the given statement is true or false, justifying your answer: $\boxed{1}$

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2.$$

(d) Indicate, with justification, whether the following statement is true or false: $\boxed{1}$

$$\frac{d}{dx}[f(g(x))] = f'(x)g'(x).$$

(e) Find all the critical values of the function $g(x) = 2\sin x + \cos^2 x$. $\boxed{2}$

Question 3 [12 marks]

3.1 Let

$$F(x) = \frac{x^3 - 1}{|x - 1|}.$$

(a) Evaluate the following limits: $\boxed{7}$

(i)

$$\lim_{x \to 1^-} F(x).$$

(ii)

$$\lim_{x \to 1^+} F(x).$$

(iii)
$$\lim_{x \to 1} F(x).$$

(b) Is the function F differentiable at $x = 1$? Explain your answer. $\boxed{2}$

3.2 Let
$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0, \\ x & \text{if } 0 \le x < 2, \\ x - 1 & \text{if } x \ge 2 \end{cases}.$$

Sketch the graph of the function indicating all the important points. $\boxed{3}$

Question 4 [5 marks]

Sketch the graph of a function g defined on $\{x|x^2 - 1 \neq 0\}$ which satisfies the following conditions:

(i) $g(0) = 0, 0 = g(x)$ only if $x = 0$.

(ii) $g(-x) = g(x)$.

(iii)

$$\lim_{x\to 1^-} g(x) = -\infty, \qquad \lim_{x\to 1^+} g(x) = \infty$$
$$\lim_{x\to -1^-} g(x) = \infty, \qquad \lim_{x\to -1^+} g(x) = -\infty$$
$$\lim_{x\to \pm\infty} g(x) = 2$$

(iv) $g'(0) = 0$.

(v) g is increasing on $(-\infty, 0)$, and decreasing elsewhere.

(vi) g is concave upward on $(-\infty, -1)$ and $(1, \infty)$, while it is concave downward on $(-1, 1)$.

Question 5 [11 marks]

5.1 Find y' given that $x^2 \cos y + \sin 2y = xy$. $\boxed{4}$

5.2 Find $\frac{dy}{dx}$ if $y = \sqrt{1 + 4 \sin x}$. $\boxed{3}$

5.3 Find an equation of the tangent line and the normal line to the curve $y = \sqrt{1 + 4 \sin x}$ at the point $(0, 1)$. $\boxed{4}$

Question 6 [9 marks]

6.1 Let $f(x) = x^3 + 5x + 4$. Find $f'(x)$ from first principles, that is, using the definition of the derivative. $\boxed{3}$

6.2 Given that $y = \sin\left(\sqrt{\cos 2x}\right)$, find y'. $\boxed{3}$

6.3 Show that

$$\frac{d}{dx}(\sec^2 x) = \frac{d}{dx}(\tan^2 x)$$

$\boxed{3}$

Question 7 [13 marks] Let

$$f(x) = \frac{x^2}{\sqrt{x^4 - 1}}$$

7.1 Check algebraically if the function f is even, odd or neither. $\boxed{2}$

7.2 Identify all possible vertical asymptotes for f showing clearly calculations verifying your answer. $\boxed{2}$

7.3 Find all possible horizontal asymptotes for f. Show clear calculations to verify your result. $\boxed{4}$

7.4 Consider the function $f(x) = x^4 - 2x^2 + 3$. At which intervals is the function increasing or decreasing? Characterise all the turning points of the function. $\boxed{5}$

Question 8 [9 marks]

8.1 Evaluate the following limit: $\boxed{4}$

$$\lim_{x \to 0} \frac{\tan 4x}{\sin 3x}.$$

8.2 Given that $f''(\theta) = \sin\theta - \cos\theta$, $f(0) = 3$, $f'(0) = 4$, find f. $\boxed{5}$

Question 9 [7 marks]

The the surface area of a cube is increasing at the rate of $\frac{4}{3}$ cm^2/min. How fast is the volume increasing when the length of an edge is 30 cm? $\boxed{5}$