



FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE **MAFT0B2/MA2BFET**
MATHEMATICS 2B FOR TEACHERS

CAMPUS **APK**
ASSESSMENT **EXAM**

DATE 26/11/2019

TIME 16:30

ASSESSOR(S)

MR T. MUDZIIRI SHUMBA

MODERATOR

MS S. RICHARDSON

DURATION 120 MINUTES

MARKS 80

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 10 PAGES

INSTRUCTIONS:

1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. CALCULATORS ARE **NOT ALLOWED**.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE FACING BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [7 marks]

For questions 1.1 – 1.4, choose **one** correct answer, and make a cross (X) in the correct block.

Question	A	B	C	D	E
1.1					
1.2					
1.3					
1.4					

1.1 Suppose that

$$\lim_{x \rightarrow 2^+} f(x) = 4.$$

If $\lim_{x \rightarrow 2} f(x)$ exists, to what value does it converge?

1

- (A) 2
- (B) -2
- (C) 4
- (D) -4
- (E) None of the above.

1.2 Consider the function given by

$$f(x) = \begin{cases} 1 & \text{if } x < -1, \\ x & \text{if } -1 \leq x < 1, \\ (x+1)^2 & \text{if } x \geq 1. \end{cases}$$

At what value(s) of x is the function discontinuous?

2

- (A) -1 only.
- (B) -1 and 1.
- (C) 1 only.
- (D) 0 and 1.
- (E) None of the above.

1.3 Let $f(x) = 4\sqrt{x}$. The value of c such that the conclusion of the mean value theorem is satisfied on the interval $[0, 25]$ is:

2

- (A) 25
- (B) $\frac{25}{4}$
- (C) $\frac{1}{5}$
- (D) 4
- (E) None of the above.

1.4 Given that $f(0) = -1$, $f'(0) = -2$, $g(0) = -3$ and $g'(0) = 4$, the slope of the tangent line to the curve $y = f(x)(g(x) + 2)$ at $x = 0$ is: 2

- (A) -2 .
- (B) 1 .
- (C) 6 .
- (D) -6 .
- (E) None of the above.

Question 2 [7 marks]

(a) State the domain of the function

$$h(t) = \frac{2}{3t + 2}.$$

1

(b) Give the domain of the function $g(x) = \sqrt{x^2 - 25}$.

2

(c) State whether the given statement is true or false, justifying your answer:

1

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2.$$

(d) Indicate, with justification, whether the following statement is true or false:

1

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x).$$

(e) Find all the intercepts of the graph of the function $y = 2 + 2 \tan x$, $0 \leq x \leq 2\pi$.

2

Question 3 [12 marks]

3.1 Let

$$F(x) = \frac{x^3 + 1}{|x + 1|}.$$

(a) Evaluate the following limits:

7

(i)

$$\lim_{x \rightarrow -1^-} F(x).$$

(ii)

$$\lim_{x \rightarrow -1^+} F(x).$$

(iii)

$$\lim_{x \rightarrow -1} F(x).$$

(b) Is the function F differentiable at $x = -1$? Explain your answer.

2

3.2 Let

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x < 0, \\ x & \text{if } 0 \leq x < 2, \\ x - 2 & \text{if } x \geq 2 \end{cases}.$$

Sketch the graph of the function indicating all the important points.

3

Question 4 [5 marks]

Sketch the graph of a function g defined on $(-\frac{1}{2}\pi, \frac{1}{2}\pi)$ which satisfies the following conditions:

(i) $g(0) = 0$.

(ii) $g(-x) = g(x)$.

(iii)

$$\lim_{x \rightarrow \frac{\pi}{2}^-} g(x) = \infty.$$

(iv)

$$\lim_{x \rightarrow -\frac{\pi}{2}^+} g(x) = \infty.$$

(v) $g'(0) = 0$.

(vi) g is increasing on $(0, \frac{1}{2}\pi)$.

(vii) g is concave upward on $(0, \frac{1}{2}\pi)$.

Question 5 [11 marks]

Given that $x^2 + xy - 4y = 2$,

5.1 Use implicit differentiation to find $\frac{dy}{dx}$.

4

5.2 Rewrite y explicitly as a function of x and hence find $\frac{dy}{dx}$.

4

5.3 Using your answers to the first and second part, find the equation of the normal line to the curve given by the equation at $x = 2$.

3

Question 6 [9 marks]

6.1 Find the derivative of the function $y = x^4 \tan(2x^2)$

3

6.2 Given that $y = \sin(\sqrt{\cos x})$, find y' .

3

6.3 Show that

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

3

Question 7 [13 marks] Let

$$f(x) = \frac{x^2}{x^2 - 1}$$

7.1 Check algebraically if the function f is even, odd or neither.

2

7.2 Identify all possible vertical asymptotes for f showing clearly calculations verifying your answer.

2

7.3 Find all possible horizontal asymptotes for f . Show clear calculations to verify your result.

4

7.4 Consider the function $f(x) = x^4 - 2x^2 + 3$. Find all the intervals of concavity, and discuss concavity in these intervals. Identify all possible inflection points. 5

Question 8 [9 marks]

8.1 Evaluate the following limit:

4

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{\tan 4x}.$$

8.2 Given that $f''(\theta) = \sin \theta + \cos \theta$, $f(0) = 3$, $f'(0) = 4$, find f .

5

Question 9 [7 marks]

Find the maximum area of a rectangle that can be inscribed in a semicircle of radius r .