UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE **ASME2A1**
SEQUENCES, SERIES AND VECTOR CALCULUS

CAMPUS **APK**

SUPPLEMENTARY EXAM **JANUARY 2020**

EXAMINER	MR M ASKES
INTERNAL MODERATOR	DR F SCHULZ
DURATION	2 HOURS
MARKS	50

SURNAME AND INITIALS ______________________

STUDENT NUMBER ______________________

IDENTITY NUMBER ______________________

NUMBER OF PAGES: 1 + 11

INSTRUCTIONS:
1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE **NOT** ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL WORKING OUT

Question 1

Determine whether the following statements are true or false. If true, give a short justification. If false, give a counter example. [10]

(1.1) If $\{a_n\}$ is a divergent sequence, then $\{a_n^2\}$ is divergent. (2)

(1.2) If $\{a_n\}$ is a convergent sequence, then $\{\frac{1}{a_n}\}$ is convergent. (2)

(1.3) The ratio test can be used to determine whether $\sum_{n=0}^{\infty} \frac{1}{n!}$ converges. (2)

(1.4) If $a_n > 0$ and $\lim_{n\to\infty}(a_{n+1}/a_n) < 1$, then $\lim_{n\to\infty} a_n = 0$. (2)

(1.5) If $a_n > 0$ and $\sum a_n$ converges, then $\sum(-1)^n a_n$ converges. (2)

Question 2

Show that the sequence defined by [4]

$$a_1 = 1 \qquad a_{n+1} = 3 - \frac{1}{a_n} \ \text{ for } \ n \geq 1$$

is increasing and bounded above by 3.

Question 3

State and prove the alternating series test. [6]

Question 4 [5]

(4.1) Test whether (3)

$$\sum_{n=1}^{\infty}\left(\frac{n^2+1}{2n^2+1}\right)^n$$

is absolutely convergent, conditionally convergent, or divergent.

(4.2) Find the (exact) sum of the series (2)

$$\sum_{n=2}^{\infty} 2\left(\frac{1}{n}-\frac{1}{n+3}\right).$$

Question 5

Find the radius and interval of convergence for [4]

$$\sum_{n=0}^{\infty}(-1)^n\frac{(x-3)^n}{2n+1}.$$

Question 6

Let k be any real number. Use the Maclaurin series of $(1+x)^k$ to obtain the Maclaurin series for [3]

$$f(x) = x^3\left(1+x^2\right)^k.$$

Question 7

Consider the curve with parametric equations $x = t^2$, $y = 1 - 3t$, $z = 1 + t^3$. Does the curve pass through the points $(1, 4, 0)$, $(9, -8, 28)$ and $(4, 7, -6)$? [3]

Question 8

Show that if $\mathbf{r}$ is a vector function such that $\mathbf{r}''$ exists, then [3]

$$\frac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}(t) \times \mathbf{r}''(t).$$

Question 9

Find the unit tangent vector, unit normal vector, and the curvature of the curve [7]

$$\mathbf{r}(t) = \langle t, 3\cos(t), 3\sin(t)\rangle .$$

Question 10

Derive the formula [3]

$$\mathbf{a} = v'\mathbf{T} + \kappa v^2 \mathbf{N}.$$

Question 11

Determine whether the following statement is true or false. Give a short justification. [2]

The binormal vector is $\mathbf{B}(t) = \mathbf{N}(t) \times \mathbf{T}(t)$.