UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE	**ASME2A1** SEQUENCES, SERIES AND VECTOR CALCULUS
CAMPUS	**APK**
EXAM	**NOVEMBER 2019**

EXAMINER	MR M ASKES
INTERNAL MODERATOR	DR F SCHULZ
DURATION	2 HOURS
MARKS	50

SURNAME AND INITIALS ______________________________

STUDENT NUMBER ______________________________

IDENTITY NUMBER ______________________________

NUMBER OF PAGES: 1 + 12

INSTRUCTIONS:

1. ANSWER ALL QUESTIONS ON THE PAPER IN PEN
2. CALCULATORS ARE **NOT** ALLOWED
3. INDICATE **CLEARLY** ANY ADDITIONAL WORKING OUT

Question 1

Determine whether the following statements are true or false. If true, give a short justification. If false, give a counter example. [10]

(1.1) A sequence, $\{a_n\}$, is bounded if there exists a real number k such that

$$a_n < k$$

for all $n \in \mathbb{N}$. (2)

(1.2) If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent. (2)

(1.3) If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ is divergent. (2)

(1.4) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} |a_n|$ is convergent. (2)

(1.5) If $\{a_n\}$ is decreasing and $a_n > 0$ for all $n \in \mathbb{N}$, then $\{a_n\}$ is convergent. (2)

Question 2

Determine whether the sequence defined as follows is convergent or divergent:

$$a_1 = 1 \quad a_{n+1} = 4 - a_n \quad \text{for } \; n \geq 1.$$

Then, what happens if the first term is $a_1 = 2$? [4]

Question 3

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. Prove that if $\sum b_n$ is divergent and $a_n \geq b_n$ for all $n \in \mathbb{N}$, then $\sum a_n$ is also divergent. [3]

Question 4 [5]

(4.1) Test whether (3)

$$\sum_{n=1}^{\infty} \frac{n^{100} 100^n}{n!}$$

is absolutely convergent, conditionally convergent, or divergent.

(4.2) Find the sum of the series (2)

$$\sum_{n=0}^{\infty} (-1)^n \frac{(2\pi)^{2n}}{(2n)!}.$$

Question 5

State the alternating series estimation theorem. [3]

Question 6

Find the radius and interval of convergence for [4]

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n4^n}.$$

Question 7

Use the Maclaurin series of $\cos(x)$ to obtain the Maclaurin series for [3]

$$f(x) = x\cos\left(\frac{x^2}{2}\right).$$

Question 8

At what points does the helix [3]

$$\mathbf{r}(t) = \langle \sin(t), \cos(t), t \rangle$$

intersect the sphere $x^2 + y^2 + z^2 = 5$?

Question 9

Show that [3]

$$\frac{d}{dt}\left[\mathbf{u}\left(f(t)\right)\right] = f'(t)\mathbf{u}'\left(f(t)\right).$$

Question 10

Find the unit tangent vector, unit normal vector, and the curvature of the curve [7]

$$\mathbf{r}(t) = \langle t, 3\cos(t), 3\sin(t) \rangle .$$

Question 11

Find the velocity, acceleration, and speed of a particle with the position function [3]

$$\mathbf{r}(t) = \langle 2\cos(t), 3t, 2\sin(t)\rangle.$$

Question 12

Determine whether the following statement is true or false. If true, give a short justification. If false, give a counter example. [2]

Different parametrizations of the same curve result in identical tangent vectors at a given point on the curve.