UNIVERSITY OF JOHANNESBURG

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE	ASMA1A1 CALCULUS OF ONE-VARIABLE FUNCTIONS
CAMPUS	APK
ASSESSMENT	SUPPLEMENTARY EXAM

DATE	JANUARY 2020
ASSESSOR(S)	MR M SAGMING MS S RICHARDSON
INTERNAL MODERATOR	DR A SWARTZ
DURATION 2 HOURS	MARKS 70

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 13 PAGES

INSTRUCTIONS: 1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [10 marks]

For questions 1.1 - 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 The statement, "there is a negative integer that is less than ten", when translated into first-order language becomes: [1]

(a) $(\exists x \in \mathbb{Z})(x < 0 \wedge x < 10)$

(b) $(\forall x \in \mathbb{Z})(x \leq 0 \wedge x < 10)$

(c) $(\exists x \in \mathbb{Z})(x < 0 \vee x \leq 10)$

(d) $(\exists x \in \mathbb{Z})(x \leq 0 \vee x < 10)$

(e) None of the above.

1.2 Which one of the following first-order statements is true? [1]

(a) $(\forall x \in \mathbb{R})(x^3 = x)$

(b) $(\forall x \in \mathbb{R})(x^2 = x)$

(c) $(\forall x \in \mathbb{R})(x^2 > x)$

(d) $(\forall x \in \mathbb{Z}^+)(x^2 \geq x)$

(e) None of the above.

1.3 Suppose $H = f \circ g \circ h$ and $H(x) = \sqrt[10]{\sqrt{x} - 3}$. Select the correct option below: [1]

(a) $g(x) = \sqrt[10]{x}$

(b) $h(x) = \sqrt[10]{x}$

(c) $f(x) = \sqrt[10]{x}$

(d) All of the above

(e) None of the above.

1.4 The exact value of $\arctan(-1)$ in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ is ... [1]

(a) $\frac{\pi}{4}$

(b) -1

(c) $3\frac{\pi}{4}$

(d) $-\frac{\pi}{4}$

(e) None of the above.

1.5 $\cosh(\ln(x)) =$ [1]

(a) $\frac{1-x^2}{2x}$

(b) $\frac{2x}{x^2-1}$

(c) $\frac{x^2+1}{2x}$

(d) $\frac{x^2-1}{2x}$

(e) None of the above.

1.6 The point $P(-3,-8)$ is on the graph of $y = f(x)$. Which point will be on the graph of $y = -f(x-5)$? [1]

(a) $(2,8)$

(b) $(-8,8)$

(c) $(-8,-8)$

(d) $(8,-8)$

(e) None of the above.

1.7 According to the Intermediate Value Theorem, if f is continuous on the closed interval $[a,b]$ and 0 is between $f(a)$ and $f(b)$, $f(a) \neq f(b)$, then there exists a number c in (a,b) such that: [1]

(a) $f(a) = f(b)$

(b) $f(c) = 0$

(c) $c = 0$

(d) $f(0) = c$

(e) None of the above.

1.8 $\dfrac{d}{dx}(x^4 \ln x^2) =$ [1]

(a) $2x^3(1 + 4\ln x)$

(b) $x^2(1 + 4\ln x^2)$

(c) $2x^3(1 + \ln x^2)$

(d) $2x^3(1 + 2\ln x^2)$

(e) None of the above.

1.9 The value of the definite integral $\displaystyle\int_1^2 \frac{x + x^3}{x^2}\, dx$ is: [1]

(a) $\frac{1}{2} + \ln 2$

(b) $\frac{3}{2} + \ln 2$

(c) $2 + \ln 2$

(d) $2\ln 2$

(e) None of the above.

1.10 If $y = xe^{20}$ then $\dfrac{dy}{dx} =$ [1]

(a) $20xe^{19} + e^{20}$

(b) xe^{20}

(c) e^{20}

(d) $20xe^{19} + xe^{20}$

(e) None of the above.

Question 2 [2 marks]

Prove the identity: $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$ [2]

Question 3 [4 marks]

Prove that for any integer n, $n^2 + n$ is even [4]

Question 4 [4 marks]

(a) Translate the following sentence into first-order language [1]

"There exists an integer whose cube equals itself"

(b) Negate your answer in (a) and leave the result in natural language [2]

(c) Is the statement in (b) true or false? [1]

Question 5 [7 marks]

(a) Use transformations to sketch the graph of $y = -\cos(x+\pi)$ within the interval $[0, \pi]$. **Show each step.** [2]

(b) Determine whether $f(x) = \dfrac{1}{\tan x}$ is even, odd or neither. [2]

(c) Graph the function of $y = \frac{\pi}{2} + \arctan\theta$ and state the domain and the range in interval notation. [3]

Question 6 [4 marks]

Let $f(x) = 2\ln(x-1) + 1$ find:

(a) the domain of $f(x)$. [1]

(b) the range of $f(x)$ [1]

(c) Sketch the graph of $f(x)$. [2]

Question 7 [2 marks]

Solve for x: $2^{\log(x+1)} = \sin\dfrac{\pi}{2}$ [2]

Question 8 [2 marks]

Evaluate the following limit. Show all steps. [2]

$$\lim_{x\to\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$$

Question 9 [4 marks]

Given:

$$f(x) = \begin{cases} 3x^2 & \text{if } x \leq 1 \\ 4 - x & \text{if } 1 < x \leq 4 \\ -1 & \text{if } x > 4 \end{cases}$$

(a) Prove by calculation that f is continuous at $x = 1$ [3]

(b) What kind of discontinuity is at $x = 4$? [1]

Question 10 [3 marks]

Compute $\lim_{x\to\infty} \dfrac{2-\cos x}{x+3}$ using the Squeeze Theorem. [3]

Question 11 [2 marks]

Use the Intermediate Value Theorem to show that the equation $x^3 + x + 1 = 0$ has a solution in the interval $[-2, 0]$. [2]

Question 12 [5 marks]

Prove the Quotient Rule, that is: [5]

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Question 13 [7 marks]

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ If $y = \sqrt{x}\ln\sqrt{x}$ [4]

(b) Find the equation of the tangent line to the curve at the given point [3]

$$x^4 + 4xy + y^4 = 6,\ (1,1)$$

Question 14 [4 marks]

Use L'Hospital's Rule to evaluate the following limit: [4]

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}$$

Question 15 [10 marks]

Evaluate the following:

(a) $\int_1^4 \frac{64 - 12x}{\sqrt{x}}\, dx$ [3]

(b) $\int_1^2 x^3\sqrt{x^2 + 1}\, dx$ [3]

(c) $\dfrac{d}{dx}\displaystyle\int_{\tan x}^{\frac{\pi}{4}} \sec t \tan t \; dt$ [4]