UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

LINEAR ALGEBRA A
ASMA2A2

SUPPLEMENTARY EXAMINATION 2019

DATE: JANUARY 2020

ASSESSOR: C MARAIS

MODERATOR: G BRAATVEDT

DURATION: 120 MINUTES **MARKS:** 50

SURNAME AND INITIALS:……………………………………………………………………

STUDENT NUMBER:……………………………………………..

CONTACT NUMBER:……………………………………………..…

NUMBER OF PAGES: 9

INSTRUCTIONS:

ANSWER ALL QUESTIONS IN PEN
SHOW NECESSARY WORKING AND CALCULATIONS
YOU MAY USE A CALCULATOR
USE THE BLANK PAGES FOR ROUGH WORK
INDICATE IF YOU WANT WORK ON BLANK PAGES TO BE MARKED
GOOD LUCK!

Question 1 - 5

Choose the correct option for the multiple choice questions below and write your answer in the table provided. [5]

Question	Answer
1.	
2.	
3.	
4.	
5.	

1. Which of the following statements is true?
   a) A scalar can be added to a vector.
   b) It is possible for the norm of a vector to equal zero even though one of its components is non-zero.
   c) The result of taking the dot product of two vectors is another vector.
   d) A vector contains magnitude and direction while a scalar does not.
   e) None of these.

2. The two vectors $(-2,1)$ and $(1,2)$ are
   a) linearly dependent of each other.
   b) orthonormal to each other.
   c) orthogonal to each other.
   d) pointing in the opposite direction of each other.
   e) None of these.

3. If $\mathbf{r}(t)=(1,0)+t(-2,1)$ is the vector equation of a line in $\mathbb{R}^2$, then $(-1,1)$ is
   a) not a point on the line.
   b) a point on the line.
   c) a vector parallel to the line.
   d) a vector perpendicular to the line.
   e) None of these.

4. Suppose $\mathbf{u}$ and $\mathbf{v}$ are vectors such that $\|\mathbf{u}+\mathbf{v}\|=\|\mathbf{u}\|+\|\mathbf{v}\|$ then
   a) $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.
   b) $\mathbf{u}$ and $\mathbf{v}$ are parallel.
   c) nothing can be said about $\mathbf{u}$ and $\mathbf{v}$.
   d) the sum of $\mathbf{u}$ and $\mathbf{v}$ is orthogonal to $\mathbf{u}$ and $\mathbf{v}$.
   e) None of these.

5. Consider the two lines with parametric equations as follows:

$$l_1 : x=-2s+1, y=s+2, z=2s+1$$

$$l_2 : x=t+1, y=-2t+2, z=2t+1$$

   Which one of the following statements is correct?
   a) l_1 and l_2 intersect in a single point.
   b) l_1 and l_2 are perpendicular.
   c) l_1 and l_2 are parallel.
   d) l_1 and l_2 are not co-planar.
   e) None of these.

Question 6 - 8

Determine whether the following statements are true or false. If true, give a short justification. If false, give a counter example.

6. A linear system of three equations in two unknowns can be inconsistent. [2]

TRUE	
FALSE	

7. A linear system with fewer equations than unknowns must have infinitely many solutions. [2]

TRUE	
FALSE	

8. If A is a square matrix such that A^2 is invertible, then A^3 is invertible. [2]

TRUE	
FALSE	

Question 9

Let $\left[A:\mathbf{b}\right]$ be the augmented matrix for a linear system $A\mathbf{x}=\mathbf{b}$ where A is a 3×3 matrix, and let $\left[R:\mathbf{d}\right]$ be the reduced row echelon from of $\left[A:\mathbf{b}\right]$. Suppose the general solution for the linear system is $\mathbf{x}=(4,0,0)+t(2,1,0)+s(5,0,1)$. Find R and $\mathbf{d}$. [3]

Question 10

Find a symmetric 2×2 matrix, A, such that $A\begin{bmatrix} 1 \\ -1 \end{bmatrix}=\begin{bmatrix} 2 \\ 3 \end{bmatrix}$. [3]

Question 11

Let A be a 4×4 matrix with $\det(A)=-3$, k a scalar and E an elementary matrix resulting from multiplying row 2 of the identity matrix, I_2, by -3. Complete the following:

a) $\det(kA^T)=$……………………

[1]

b) $\det(EA^{-1})=$……………………

[1]

c) $\det(-E^{-1})=$……………………

[1]

Question 12

Find all the values of a so that the vector $\mathbf{b}=(0,a,2)$ can be written as a linear combination of $\mathbf{u}=(1,2,3)$ and $\mathbf{v}=(2,-1,4)$. [3]

Question 13

Let $\mathbf{u}=(u_1,u_2)$ and $\mathbf{v}=(v_1,v_2)$ be vectors in $\mathbb{R}^2$. Given vector addition defined as $\mathbf{u}\oplus\mathbf{v}=(u_1-v_1,u_2-v_2)$ and the usual scalar multiplication, show that this does not define a vector space. [3]

Question 14

Let $W=\{\mathbf{p}\in\mathcal{P}_3:\mathbf{p}(0)=1\}$ be a subset of the vector space containing all polynomials of degree 3 or less. Show that W is not a subspace of $\mathcal{P}_3$. [2]

Question 15

Given that $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ form a linearly independent set, show that the set $\{\mathbf{u}+\mathbf{v}+\mathbf{w}, 3\mathbf{v}-\mathbf{w}, 2\mathbf{w}\}$ is linearly independent. [3]

Question 16

Let $\mathcal{P}_2$ be the vector space consisting of all polynomial of degree 2 or less. Consider $S=\{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4\}\subset\mathcal{P}_2$ where $\mathbf{p}_1(x)=-1+x+2x^2$, $\mathbf{p}_2(x)=x+3x^2$, $\mathbf{p}_3(x)=1+2x+8x^2$ and $\mathbf{p}_4(x)=1+x+x^2$.

a) Find a basis, B, for $\mathcal{P}_2$ consisting of vectors in S. [3]

b) For each vector in S that is not in B, find the coordinate vector with respect to the basis B. [2]

Question 17

Let $B=\{(1,2),(3,4)\}$ and $C=\{(7,3),(4,2)\}$ be two bases for $\mathbb{R}^2$. Find the change of basis matrix $P_{B\to C}$. [2]

Question 18

Let $A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 0 & 3 & 8 & 7 \\ 0 & 0 & 4 & 2 \end{bmatrix}$.

a) Find the reduced row echelon form of A. [2]

b) Find a basis for the row space of A. [1]

c) Find a basis for the null space of A. [2]

Question 19

Prove the following theorems:

a) If $S=\{\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_n\}$ is a basis for a vector space V, then every vector $\mathbf{v}$ in V can be expressed in the form $\mathbf{v}=c_1\mathbf{v}_1+c_2\mathbf{v}_2+\ldots+c_n\mathbf{v}_n$ in exactly one way. [3]

b) Let S be a nonempty set of vectors in a vector space V. If S is a linearly independent set and if $\mathbf{v}$ is a vector in V that is outside of the span of S, then the set $S\cup\{\mathbf{v}\}$ that results by inserting $\mathbf{v}$ into S is still linearly independent.[4]