UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

LINEAR ALGEBRA A
ASMA2A2

EXAMINATION 2019

DATE:	**NOVEMBER 2019**	**SESSION: 8:30 - 10:30**
ASSESSOR:	**C MARAIS**	
MODERATOR:	**G BRAATVEDT**	
DURATION:	**120 MINUTES**	**MARKS: 50**

SURNAME AND INITIALS:……………………………………………………………………

STUDENT NUMBER:……………………………………………..

CONTACT NUMBER:……………………………………………..…

NUMBER OF PAGES: **9**

INSTRUCTIONS:

ANSWER ALL QUESTIONS IN PEN
SHOW NECESSARY WORKING AND CALCULATIONS
YOU MAY USE NOT A CALCULATOR
USE THE BLANK PAGES FOR ROUGH WORK
INDICATE IF YOU WANT WORK ON BLANK PAGES TO BE MARKED
GOOD LUCK!

Question 1 - 5

Choose the correct option for the multiple choice questions below and write your answer in the table provided. [5]

Question	Answer
1.	
2.	
3.	
4.	
5.	

1. Suppose $A=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, then using cofactor expansion along the first row, we have
   a) $\det(A)=aei+afh+bdi+bfg+cdh+ceg$
   b) $\det(A)=aei+afh-bdi-bfg+cdh+ceg$
   c) $\det(A)=aei-afh-bdi+bfg+cdh-ceg$
   d) $\det(A)=aei-afh+bdi-bfg+cdh-ceg$
   e) None of these.

2. Suppose A is reduced to $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 6 \end{bmatrix}$ by first subtracting 3 times row 2 from row 3, then interchanging row 1 and row 2, and then dividing row 3 by -5. What is the determinant of A?

   a) -6 b) $6/5$ c) 30 d) $-6/5$ e) None of these.

3. Suppose $A=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ such that $\det(A)=0$, then the linear system $A\mathbf{x}=\mathbf{b}$ can
   a) only have no solutions.
   b) only have infinitely many solutions.
   c) only have a unique solution.
   d) have no solutions or infinitely many solutions.
   e) None of these.

4. Which of the following statements is correct for $n\times n$ matrices A and B?
   a) $\det(A^{-1})=-\det(A)$
   b) $\det(A^{-1})=-\dfrac{1}{\det(A)}$
   c) $\det(A+B)=\det(A)+\det(B)$
   d) $\det(3A)=3\det(A)$
   e) None of these.

5. If A is a lower triangular matrix then
   a) $\det(A)$ is the sum of the entries on the main diagonal.
   b) $\det(A)=0$ if there is a zero entry on the main diagonal.
   c) $\det(A)$ is the product of the non-zero entries of A.
   d) None of these.

Question 6 - 8

Determine whether the following statements are true or false. Give a short justification if true or counter example when false.

6. If the augmented matrix of a linear system $A\mathbf{x} = \mathbf{b}$ is row equivalent to the identity matrix, then the system must be consistent. [2]

TRUE	
FALSE	

7. If the system $A\mathbf{x} = \mathbf{b}$ has a unique solution for all $\mathbf{b}$, then A must be an invertible matrix. [2]

TRUE	
FALSE	

8. If A and B are square matrices such that $AB = B$, then A is the identity matrix. [2]

TRUE	
FALSE	

Question 9

Determine whether the lines $\mathbf{r}_1(t) = (1,1,1) + t(1,2,-1)$ and $\mathbf{r}_2(t) = (3,2,1) + t(-1,-5,3)$ are parallel, intersect, or neither. [3]

Question 10

Suppose $\mathbf{u}$ and $\mathbf{v}$ are orthogonal vectors such that $\|\mathbf{u}\| = 8$ and $\|\mathbf{v}\| = 3$. Find $\|\mathbf{u} - 2\mathbf{v}\|$. [3]

Question 11

Let U and W be subspaces of a vector space V. Show that the union, $U \bigcup W$, is not closed under vector addition.
Hint: You may use the method of Proof by Contradiction. [3]

Question 12

Consider $\mathcal{M}_{22}$, the vector space consisting of all 2×2 matrices and let $W=\left\{A\in\mathcal{M}_{22} : A=\begin{bmatrix} a & b \\ c & -a \end{bmatrix},\ a,b,c\in\mathbb{R}\right\}$.

a) Show that W is a subspace of $\mathcal{M}_{22}$. [3]

b) What is the dimension of W? Explain. [2]

Question 13

Find the values of h for which the set of vectors, $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$ are linearly independent where $\mathbf{v}_1=(1,0,0)$, $\mathbf{v}_2=(h,1,-h)$ and $\mathbf{v}_3=(1,2h,3h+1)$. [3]

Question 14

Let $\mathcal{P}_3$ be the vector space consisting of all polynomials of degree 3 or less. Consider $S=\{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4\}\subset\mathcal{P}_3$ where $\mathbf{p}_1(x)=1+3x+2x^2-x^3$, $\mathbf{p}_2(x)=x+x^3$, $\mathbf{p}_3(x)=x+x^2-x^3$ and $\mathbf{p}_4(x)=3+8x+8x^3$. Find a basis, B, for $\operatorname{span}(S)$ consisting of vectors in S. [3]

Question 15

Let $B=\{(1,2),(3,4)\}$ and $C=\{(7,3),(4,2)\}$ be two bases for $\mathbb{R}^2$ and let $\mathbf{v}=(1,0)$.

a) Find find the coordinate vectors $[\mathbf{v}]_B$ and $[\mathbf{v}]_C$. [3]

b) Find the change of basis matrix $P_{B \to C}$. [2]

c) Use the matrix in b) to compute $\left[\mathbf{v}\right]_C$ and compare your answer in a). [1]

Question 16

Let $A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

a) Find the reduced row echelon form of A. [2]

b) Find a basis for the column space of A. [1]

c) Find a basis for the null space of A. [2]

Question 17

Prove the following theorems:

a) A set S with two or more vectors is linearly dependent if and only if at least one of the vectors in S is expressible as a linear combination of the other vectors in S. [4]

b) If $\mathbf{v}$ is a vector in S that is expressible as a linear combination of other vectors in S, and if $S-\{\mathbf{v}\}$ denotes the set obtained by removing $\mathbf{v}$ from S, then S and $S-\{\mathbf{v}\}$ span the same space.
[4]