UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

SEQUENCES, SERIES AND VECTOR CALCULUS
ASMA2A1

EXAMINATION 2019

DATE:	**NOVEMBER 2019**	
ASSESSOR:	**C MARAIS**	
MODERATOR:	**M ASKES**	
DURATION:	**120 MINUTES**	**MARKS: 50**

SURNAME AND INITIALS:……………………………………………………………………

STUDENT NUMBER:……………………………………………..

CONTACT NUMBER:…………………………………………..…

NUMBER OF PAGES: **8**

INSTRUCTIONS:

ANSWER ALL QUESTIONS IN PEN
SHOW NECESSARY WORKING AND CALCULATIONS
YOU MAY USE A CALCULATOR
USE THE BLANK PAGES FOR ROUGH WORK
INDICATE IF YOU WANT WORK ON BLANK PAGES TO BE MARKED
GOOD LUCK!

Question 1 - 5

Choose the correct option for the multiple choice questions below and write your answer in the table provided [5]

Question	Answer
1.	
2.	
3.	
4.	
5.	

1. Suppose a_n and b_n are sequences such that $0 < a_n \leq b_n$. Which of the following statements are true?
   a) If b_n is convergent, then so is a_n.
   b) If b_n is divergent, then so is a_n.
   c) If $\lim_{n\to\infty} b_n = 0$ then $\lim_{n\to\infty} a_n = 0$.
   d) If $\lim_{n\to\infty} b_n = 1$ then $\lim_{n\to\infty} a_n = 1$.
   e) None of these.

2. Suppose the series $\sum a_n$ is conditionally convergent. Choose the true statement:
   a) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$
   b) $\sum |a_n|$ is convergent.
   c) a_n must be negative for infinitely many n.
   d) $\lim_{n\to\infty} a_n \neq 0$
   e) None of these.

3. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}\left(1+\frac{1}{n^2}\right)$ is
   a) bounded but divergent.
   b) unbounded and divergent.
   c) absolutely convergent.
   d) conditionally convergent.
   e) None of these.

4. Suppose the power series $\sum_{n=0}^{\infty} c_n (x-3)^n$ is convergent at $x = 5$. Then
   a) the power series diverges at $x = 0$.
   b) the power series diverges at $x = 1$.
   c) the power series converges at $x = 1$.
   d) the power series converges at $x = 2$.
   e) None of these.

5. The sum of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{n!}$ is
   a) e^{-x^2}
   b) $\cos x$
   c) $1 - e^{-x^2}$
   d) $e^{-x^2} - 1$
   e) None of these.

Question 6 - 9

Determine whether the following statements are true or false. If true, give a short justification. If false, give a counter example.

6. A sequence, a_n, is bounded if and only if there exists a real number k such that $|a_n| < k$ for all $n \in \mathbb{N}$. [2]

TRUE	
FALSE	

7. The constant sequence, $a_n = c$, for all $n \in \mathbb{N}$ and some $c \in \mathbb{R}$ converges to c [2]

TRUE	
FALSE	

8. Every unbounded sequence is monotone. [2]

TRUE	
FALSE	

9. If a_n is a divergent sequence, then $|a_n|$ is also a divergent sequence. [2]

TRUE	
FALSE	

Question 10

Find the sum of the series $\sum_{n=2}^{\infty} \frac{6}{n(n+3)} = \sum_{n=2}^{\infty} \frac{2}{n} - \frac{2}{(n+3)}$. [4]

Question 11

Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^2}{\sqrt{n}\ln n}$ is absolutely convergent, conditionally convergent, or divergent. [4]

Question 12

Consider the series $f\left(x\right)=\sum_{n=0}^{\infty}\frac{n+1}{3^{n+1}}x^{n}$ and answer the questions below:

a) Find the radius and interval of convergence of the given series. [5]

b) Find the power series representation of $f'\left(x\right)$ and its radius of convergence. [2]

Question 13

Find an equation for the tangent line to the curve $\mathbf{r}(t)=\left\langle e^t, te^t, t^2e^t\right\rangle$ at the point (e,e,e). [3]

Question 14

a) Consider the curve $\mathbf{r}(t)=\left\langle e^{-t}\cos(2t), e^{-1}\sin(2t)\right\rangle$. Calculate the arc length of the curve between the points $(1,0)$ and $(e^{\pi},0)$ [4]

b) Let $a > b > 0$. Find the curvature of the ellipse $\mathbf{r}(t) = \langle a\cos t, b\sin t\rangle$ at the point $(a,0)$. [4]

Question 15

a) Find the unit tangent vector, $\mathbf{T}$, and the principle unit normal vector, $\mathbf{N}$, of the curve with parametrisation $\mathbf{r}(t) = \langle 3\sin t, 3\cos t, 4t\rangle$. [2]

b) Find the tangential component of the acceleration vector of a particle with position function $\mathbf{r}(t) = \left\langle 2t, t^2, \frac{1}{3}t^3 \right\rangle$. [2]

Question 16

a) State and prove the Direct Comparison Test for convergent series. [4]

b) Let C be a smooth curve defined by the vector function $\mathbf{r}$. Prove that the curvature κ of C is given by the formula

$$\kappa = \frac{\left|\mathbf{r}'(t) \times \mathbf{r}''(t)\right|}{\left|\mathbf{r}'(t)\right|^3}.$$ [4]