UNIVERSITY OF JOHANNESBURG

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE	ASMA1A1 CALCULUS OF ONE-VARIABLE FUNCTIONS
CAMPUS	APK
ASSESSMENT	NOVEMBER EXAM

DATE 18/11/2019	TIME 16:30
ASSESSOR(S)	MR M SAGMING MS S RICHARDSON
INTERNAL MODERATOR	DR A SWARTZ
DURATION 2 HOURS	MARKS 70

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 12 PAGES

INSTRUCTIONS: 1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [10 marks]

For questions 1.1 - 1.10, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					
1.9					
1.10					

1.1 The negation of the following quantified statement $(\forall x \in \mathbb{R})(x > 0 \to x^2 > x)$ is: (1)

(a) $(\exists x \in \mathbb{R})(x > 0 \to x^2 < x)$

(b) $(\exists x \in \mathbb{R})(x < 0 \wedge x^2 \geq x)$

(c) $(\exists x \in \mathbb{R})(x < 0 \vee x^2 > x)$

(d) $(\exists x \in \mathbb{R})(x > 0 \wedge x^2 \leq x)$

(e) None of the above

1.2 Which one of the following first-order statements is true? (1)

(a) $(\forall x \in \mathbb{R})(x^2 = x)$

(b) $(\exists x \in \mathbb{R})(x^2 = x)$

(c) $(\forall x \in \mathbb{R})(x^2 > x)$

(d) $(\forall x \in \mathbb{Z}^+)(x^2 > x)$

(e) None of the above

1.3 The expansion of $\sum_{k=3}^{5}(-1)^k \frac{2^k}{k}$ is: (1)

(a) $-\frac{7}{4} + 3 - \frac{34}{6}$

(b) $-\frac{8}{3} + 4 - \frac{32}{5}$

(c) $\frac{9}{2} - 2 + \frac{32}{2}$

(d) $4 - \frac{3}{8} + 4$

(e) None of the above

1.4 $1 + \cot^2\theta =$ (1)

(a) $\csc^2\theta$

(b) $\sec^2\theta$

(c) $\tan^2\theta$

(d) $1 - \sin^2\theta$

(e) None of the above

1.5 Solving $|x + 5| \geq 7$ yields (1)

(a) $x \leq -12$ or $x \geq 2$

(b) $-12 \leq x \leq 2$

(c) $x \leq -10$ or $x \geq 4$

(d) $x \leq 14$ or $x \geq -6$

(e) None of the above

1.6 If $f(x) = x^3 - 1$, $g(x) = \sqrt[3]{x^2 - 1}$ and $h(x) = \sqrt{x + 2}$, then $(f \circ g \circ h)(x)$ equals: (1)

(a) 1

(b) -1

(c) x

(d) $\sqrt[5]{(x^3 - 1)^2 - 1}$

(e) None of the above

1.7 Find the limit: $\lim_{x \to 0} \left(\frac{\sin \pi}{\pi} \right)$ (1)

(a) 1

(b) 0

(c) ∞

(d) π

(e) None of these

1.8 If $y = 5\ln(5x)$ then $\dfrac{dy}{dx} =$ (1)

(a) $\frac{25}{x}$

(b) $\frac{5}{x}$

(c) $\frac{1}{x}$

(d) $\frac{1}{5x}$

(e) None of the above

1.9 $\dfrac{d^2}{dx^2}(xe^x) =$ (1)

(a) $(x+1)e^x$

(b) $(2x+1)e^x$

(c) $(x+2)e^x$

(d) $2xe^x$

(e) None of the above

1.10 The value of the definite integral $\displaystyle\int_1^{25} \frac{2}{\sqrt{x}}\,dx$ is: (1)

(a) 16

(b) 4

(c) 8

(d) 32

(e) None of the above

Question 2 [9 marks]

(a) If f is the function defined below, determine whether f is even, odd or neither:

$$f(x) = 2 - 3\cos x$$

(2)

(b) Solve for x:

$$\frac{x}{x^2+2x-3} \leq -\frac{2}{x^2+2x-3} \tag{3}$$

(c) Determine the inverse of $f(x) = 3\ln(1-e^x)$ **and find the domain of** $f^{-1}(x)$**.** (4)

Question 3 [4 marks]

Prove that $n^2 - n + 5$ is odd for all integers n.

Question 4 [4 marks]

(a) Translate the following sentence into first-order language (1)

"The cube of any real number is less than its square"

(b) Negate your answer in (a) and leave the result in natural language (2)

(c) Is the statement in (b) true or false? (1)

Question 5 [8 marks]

Determine:

(a) $\lim_{x \to -2} \dfrac{x^4 - 16}{x + 2}$ (2)

(b) $\lim_{x \to -\infty} \dfrac{\sqrt{4x^6 - x}}{x^3 + 5}$ (3)

(c) $\lim_{x\to 0} x^4 \cos\left(\frac{3}{x}\right)$ (3)

Question 6 [6 marks]

(a) Sketch

$$f(x) = \begin{cases} e^{-x} & \text{if} \quad x < 0 \\ \tan x & \text{if} \quad 0 \leq x \leq \pi \end{cases}$$

(3)

(b) Show by calculation that $f(x)$ is discontinuous at $x = 0$. (2)

(c) Is $f(x)$ differentiable at $x = 0$? Explain. (1)

Question 7 [3 marks]

If $f(x) = \dfrac{1}{3\sqrt{x}}$ determine $f'(x)$ by making use of first principles.

Question 8 [7 marks]

(a) Find the equation of the tangent line to the curve at the given point (3)

$$x^3 + 3xy + y^3 = 5,\ (1, 1)$$

(b) Find $\dfrac{dy}{dx}$ if $y = \dfrac{x^{\frac{3}{7}}(4x-1)^5(3x-2)^7}{\sqrt{x^2-1}}$ by using logarithmic differentiation. (4)

Question 9 [4 marks]

Use L'Hospital's Rule to evaluate the following limit:

$$\lim_{x\to\infty} (x^2+9)^{1/x^2}$$

Question 10 [10 marks]

Evaluate the following:

(a) $\displaystyle\int_1^4 \frac{y-\sqrt{y}}{y^2}\,dy$ (3)

(b) $\displaystyle\int_1^2 x\sqrt{x-1}\,dx$ (3)

(c) $\dfrac{d}{dx}\displaystyle\int_{\sec x}^{\frac{\pi}{4}} \tan t \; dt$ (4)

Question 11 [5 marks]

Prove the Product Rule, that is:

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$