



**FACULTY OF SCIENCE**

**DEPARTMENT OF MATHEMATICS AND APPLIED  
MATHEMATICS**

**DISCRETE MATHEMATICS FOR IT**

**ASMA2A4**

**SUPPLEMENTARY EXAMINATION 2019**

**DATE:** JANUARY 2020

**ASSESSOR:** C MARAIS

**MODERATOR:** S RICHARDSON

**DURATION:** 120 MINUTES

**MARKS: 50**

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SURNAME AND INITIALS:.....

STUDENT NUMBER:.....

CONTACT NUMBER:.....

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**NUMBER OF PAGES:** 9

**INSTRUCTIONS:**

ANSWER ALL QUESTIONS IN PEN  
SHOW NECESSARY WORKING AND CALCULATIONS  
YOU MAY USE A CALCULATOR  
USE THE BLANK PAGES FOR ROUGH WORK  
INDICATE IF YOU WANT WORK ON BLANK PAGES TO BE MARKED  
GOOD LUCK!

Question 1

Consider a propositional language where

- $p$  means “Aldo is Italian”
- $q$  means “Bob is English”

Formalise the following sentences writing them as propositional formula using logical connectives:

a) “Aldo is Italian or if Aldo isn’t Italian then Bob is English.” [1]

b) “Either Aldo is Italian and Bob is English, or neither Aldo is Italian nor Bob is English.” [1]

c) Negate the formula in B) and write it in negation normal form. [2]

Question 2

Use the truth table below to verify whether the following logical equivalence holds:  $(p \wedge q) \vee r \equiv (p \rightarrow \neg q) \rightarrow r$  [3]

$p$	$q$	$r$	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

$\therefore$

Question 3

Use Semantic Tableau to verify whether the following formula is valid:

[4]

$$\exists x(P(x) \vee Q(x)) \rightarrow (\exists x P(x) \vee \exists x Q(x))$$

Question 4

Write the following formula in prenex conjunctive normal form:

[4]

$$\exists z \left( \exists x Q(x, z) \vee \exists x P(x) \right) \rightarrow \neg \left( \neg \exists x P(x) \wedge \forall x \exists z Q(z, x) \right)$$

Question 5

- a) An automobile manufacturer has four colours available for automobile exteriors and three for interiors. How many different color combinations can he produce?

[1]

- b) A lady wishes to colour her fingernails on one hand using exactly two of the colours red, yellow, and blue. How many ways can she do this?

[2]

Question 6

What is the coefficient of  $x^3y^4$  in  $(5x - 2y)^7$ ?

[3]

Question 7

How many cards must be drawn from a standard deck of cards consisting of 52 cards (13 of each of 4 suits) to be sure that you draw at least two cards of the same suit?

[1]

Question 8

At a university, 14 students signed up for Discrete Mathematics, 12 students signed up for Linear Algebra, and 6 students signed up for both courses. How many of the students signed up for at least one of the two courses?

[3]

Question 9

Find the remainder when  $11^{2402}$  is divided by 3000.

[3]

Question 10

- a) Use Euclid's algorithm to find  $\gcd(684, 589)$  and write this as a linear combination, i.e. find  $u$  and  $v$  such that  $\gcd(684, 589) = 684u + 589v$ .

[4]

- b) Find the general solution of the Diophantine equation,  $598x - 684y = 247$ ,  $x, y \in \mathbb{Z}$ .

[3]

c) Solve the linear congruence,  $589x \equiv 247 \pmod{684}$ .

[2]

Question 11

Use the Chinese Remainder Theorem to solve the following system of congruences:

[4]

$$x \equiv 3 \pmod{5}$$

$$x \equiv 4 \pmod{6}$$

$$x \equiv 5 \pmod{7}$$

Question 12

Consider an RSA cryptosystem with  $n = 421 \times 401$  and  $e = 247$ . Give (but do not solve) the congruence that you would use to find a decode exponent  $d$ . Simplify this congruence where possible. [2]

Question 13

Prove the following theorems:

- a) Every natural number greater than 1 has a prime divisor.

[3]



b) There are infinitely many prime numbers.

[4]