UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

DISCRETE MATHEMATICS FOR IT

ASMA2A4

EXAMINATION 2019

DATE:	**NOVEMBER 2019**
ASSESSOR:	**C MARAIS**
MODERATOR:	**S RICHARDSON**
DURATION:	**120 MINUTES** — **MARKS: 50**

SURNAME AND INITIALS:...

STUDENT NUMBER:...

CONTACT NUMBER:...

NUMBER OF PAGES: **9**

INSTRUCTIONS:
ANSWER ALL QUESTIONS IN PEN
SHOW NECESSARY WORKING AND CALCULATIONS
YOU MAY USE A CALCULATOR
USE THE BLANK PAGES FOR ROUGH WORK
INDICATE IF YOU WANT WORK ON BLANK PAGES TO BE MARKED
GOOD LUCK!

Question 1

Consider a propositional language where

- p means “Paula is happy”
- q means “Paula paints a picture”
- r means “Renzo is happy”.

Formalise the following sentences writing them as propositional formula using logical connectives:

a) “If Paula is happy and paints a picture, then Renzo isn’t happy.” [1]

b) “Paula is happy only if she paints a picture or Renzo is happy.” [1]

c) Negate the formula in a) and write it in negation normal form. [2]

Question 2

Use the truth table below to verify whether the following logical argument is sound: $p \vee (\neg q \wedge r) \vDash q \vee \neg r \rightarrow p$ [3]

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

$\therefore$

Question 3

Use Semantic Tableau to verify whether the following argument form is valid: [4]

$$\exists x\left(P(x) \to Q(x)\right);\ \exists x P(x) \vDash \exists x Q(x)$$

Question 4

Write the following formula in prenex conjunctive normal form: [4]

$$\exists z\left(\exists x Q(x,z) \vee \exists x P(x)\right) \rightarrow \neg\left(\neg \exists x P(x) \wedge \forall x \exists z Q(z,x)\right)$$

Question 5

a) In a digital computer, a bit is one of the integers $\{0,1\}$, and a word is any string of 32 bits. How many different words are possible? [1]

b) A die is rolled 30 times. What is the probability that a 6 turns up exactly 5 times? [2]

Question 6

What is the coefficient of $x^4 y^3$ in $(5x+2y)^7$? [3]

Question 7

Two people share the same birthday if it is on the same day and month of the year. If we ignore leap days, how many people need to be in a group to guarantee that at least two people will share the same birthday? [1]

Question 8

At a university, 14 students signed up for Discrete Mathematics, 12 students signed up for Linear Algebra, and 6 students signed up for both courses. How many of the students signed up for at least one of the two courses? [2]

Question 9

Find the general solution of the Diophantine equation, $33x - 21y = 15, \quad x, y \in \mathbb{Z}$. [4]

Question 10

a) Use Euclid's algorithm to find $\gcd(381, 408)$ and write this as a linear combination, i.e. find u and v such that $\gcd(381, 408) = 381u + 408v$. [4]

b) Solve the following congruence or explain why no solution exists: $381x \equiv 231 \pmod{408}$. [3]

Question 11

Use the Chinese Remainder Theorem to solve the following system of congruences: [4]

$$x \equiv 2 \pmod{5}$$
$$x \equiv 4 \pmod{7}$$
$$x \equiv 1 \pmod{8}$$

Question 12

Consider an RSA cryptosystem with $n = 11 \times 13 = 143$ and $e = 7$, i.e. the public key is (143; 7). Determine d in the corresponding private key (143; d). [4]

Question 13

Prove the following theorems:

a) Every natural number greater than 1 has a prime divisor. [3]

b) There are infinitely many prime numbers. [4]