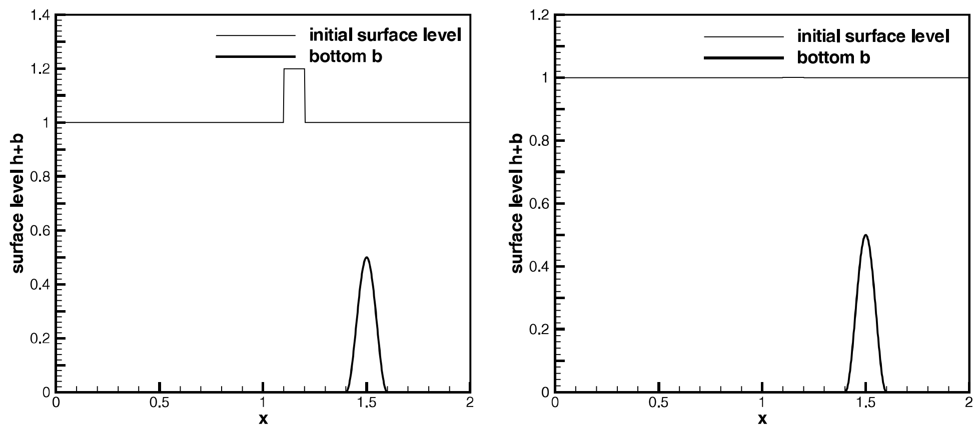

Fig. 2. The initial surface level $h + b$ and the bottom b for a small perturbation of a steady-state water. Left: a big pulse $\epsilon = 0.2$; right: a small pulse $\epsilon = 0.001$.

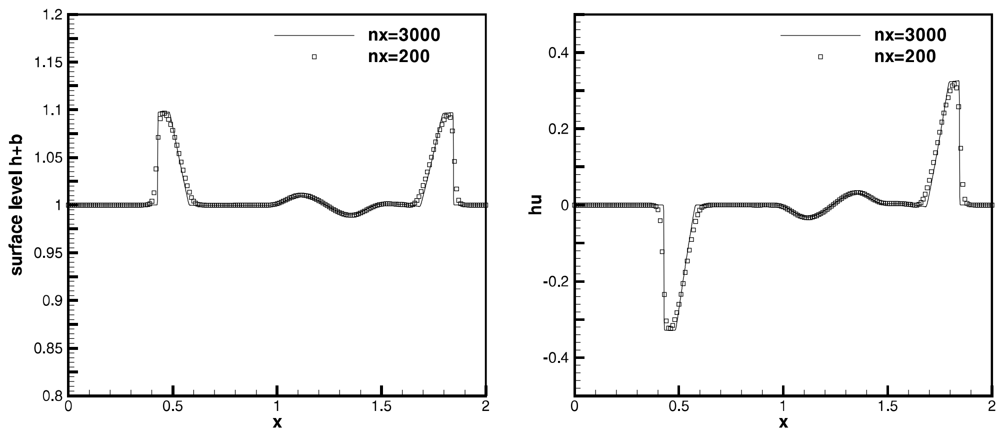

Fig. 3. Small perturbation of a steady-state water with a big pulse. $t = 0.2$ s. Left: surface level $h + b$; right: the discharge hu.

and right at the characteristic speeds $\pm\sqrt{gh}$. Many numerical methods have difficulty with the calculations involving such small perturbations of the water surface [12]. Both sets of initial conditions are shown in Fig. 2. The solution at time $t = 0.2$ s for the big pulse $\epsilon = 0.2$, obtained on a 200 cell uniform grid with simple transmissive boundary conditions, and compared with a 3000 cell solution, is shown in Fig. 3. The one for the small pulse $\epsilon = 0.001$ is shown in Fig. 4. For this small pulse problem, we take $\varepsilon = 10^{-9}$ in the WENO weight formula (2.2), such that it is smaller than the square of the perturbation. At this time, the downstream-traveling water pulse has already passed the bump. In the figures, we can clearly see that there are no spurious numerical oscillations, verifying the essentially nonoscillatory property of the modified WENO-LF scheme.

4.4. The dam breaking problem over a rectangular bump

In this example we simulate the dam breaking problem over a rectangular bump, which involves a rapidly varying flow over a discontinuous bottom topography. This example was used in [21].

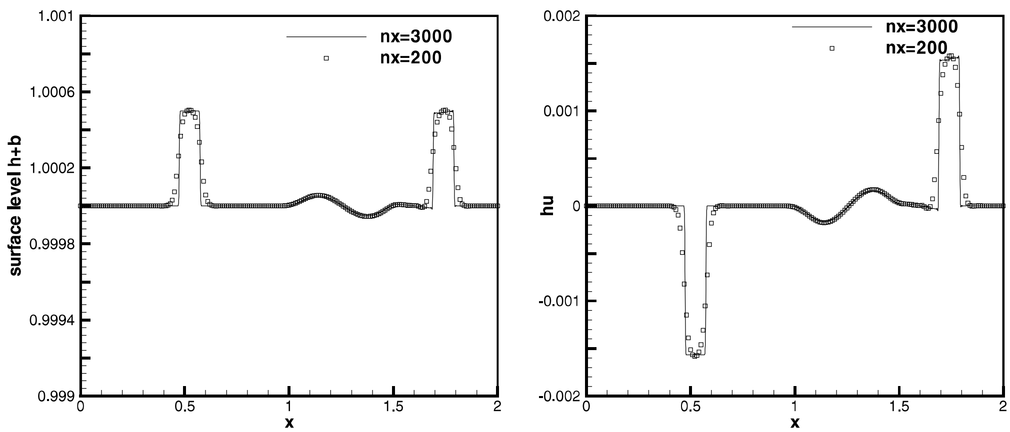

Fig. 4. Small perturbation of a steady-state water with a small pulse. $t = 0.2$ s. Left: surface level $h + b$; right: the discharge hu.

The bottom topography takes the form:

$$b(x) = \begin{cases} 8 & \text{if } |x - 750| \leqslant 1500/8, \\ 0 & \text{otherwise} \end{cases} \tag{4.5}$$

for $x \in [0, 1500]$. The initial conditions are

$$(hu)(x,0) = 0 \quad \text{and} \quad h(x,0) = \begin{cases} 20 - b(x) & \text{if } x \leqslant 750, \\ 15 - b(x) & \text{otherwise.} \end{cases} \tag{4.6}$$

The numerical results with 500 uniform points (and a comparison with the results using 5000 uniform points) are shown in Figs. 5 and 6, with two different ending time $t = 15$ s and $t = 60$ s. In this example, the water height $h(x)$ is discontinuous at the points $x = 562.5$ and $x = 937.5$, while the surface level $h(x) + b(x)$ is smooth there. Our scheme works well for this example, giving well resolved, nonoscillatory solutions using 500 points which agree with the converged results using 5000 points.

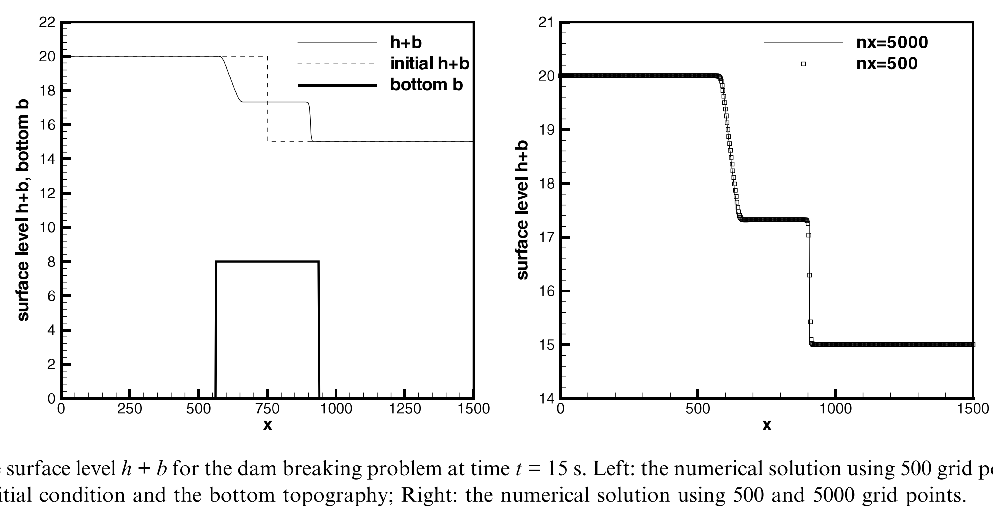

Fig. 5. The surface level $h + b$ for the dam breaking problem at time $t = 15$ s. Left: the numerical solution using 500 grid points, plotted with the initial condition and the bottom topography; Right: the numerical solution using 500 and 5000 grid points.

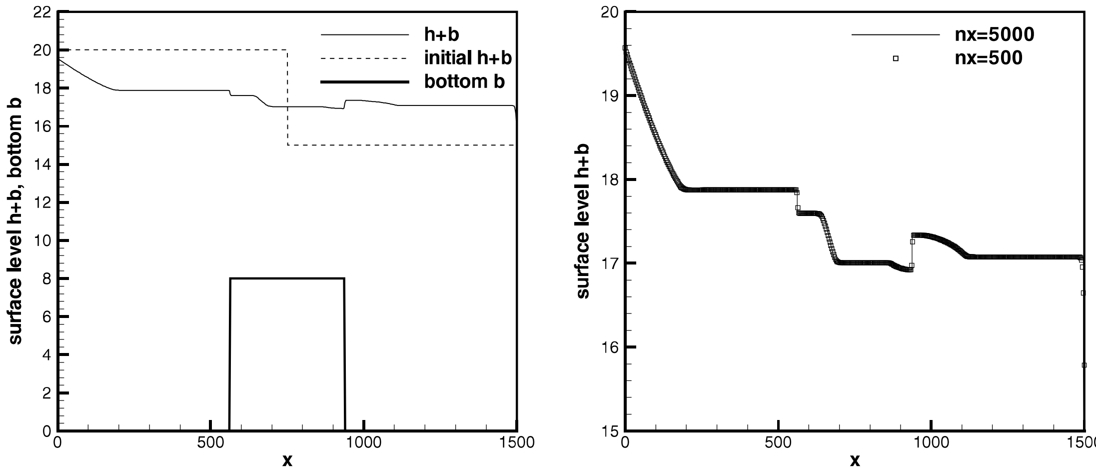

Fig. 6. The surface level $h + b$ for the dam breaking problem at time $t = 60$ s. Left: the numerical solution using 500 grid points, plotted with the initial condition and the bottom topography; Right: the numerical solution using 500 and 5000 grid points.

4.5. Steady flow over a hump

The purpose of this test case is to study the convergence in time towards steady flow over a bump. These are classical test problems for transcritical and subcritical flows, and they are widely used to test numerical schemes for shallow water equations. For example, they have been considered by the *working group on dam break modeling* [5], and have been used as a test case in, e.g., [20].

The bottom function is given by

$$b(x) = \begin{cases} 0.2 - 0.05(x-10)^2 & \text{if } 8 \leqslant x \leqslant 12, \\ 0 & \text{otherwise} \end{cases} \tag{4.7}$$

for a channel of length 25 m. The initial conditions are taken as

$$h(x,0) = 0.5 - b(x) \quad \text{and} \quad u(x,0) = 0.$$

Depending on different boundary conditions, the flow can be subcritical or transcritical with or without a steady shock. The computational parameters common for all three cases are: uniform mesh size $\Delta x = 0.125$ m, ending time $t = 200$ s. Analytical solutions for the various cases are given in [5].

(a) Transcritical flow without a shock.

- upstream: The discharge $hu = 1.53$ m^3/s is imposed.
- downstream: The water height $h = 0.66$ m is imposed when the flow is subcritical.

The surface level $h + b$ and the discharge hu, as the numerical flux for the water height h in Eq. (1.1), are plotted in Fig. 7, which show very good agreement with the analytical solution. The correct capturing of the discharge hu is usually more difficult than the surface level $h + b$, as noticed by many authors. In Fig. 8, we compare the pointwise errors of the numerical solutions obtained with 200 and 400 uniform grid points. The convergence history, measured by the L^1 norm of the residue, is given in Fig. 9, left.