UNIVERSITY OF JOHANNESBURG

APPLIED MATHEMATICS 1B
APM01B1/APM1B10

Introduction to Dynamics

SUPPLEMENTARY EXAMINATION

January 2020

Duration: 2 hours **Marks:** 50

Assessor: Prof CM Villet
Mr KD Anderson
Moderator: Dr GJ Kemp

Student number: | | | | | | | | | |

Initials & surname: ____________________

Instructions:

1. Answer all the questions.
2. All calculations must be shown.
3. Pocket calculators are permitted, work to at least three significant figures throughout.
4. All quantities are given in SI units.
5. All symbols have their usual meaning and all angles are measured in radians (unless stated otherwise).
6. Vectors are indicated throughout by bold face type, for example $\boldsymbol{a}$, $\boldsymbol{r}$.
7. Useful results are supplied at the end of this paper and can be considered as given.

Question	Marks	Score
1	10	
2	10	
3	10	
4	8	
5	12	
Total:	50	

Question 1 [10 marks]

A charged particle with mass m and charge q moves at a constant velocity $\boldsymbol{v}_0$ when an electric field is introduced perpendicular to its direction of motion. The field varies periodically with respect to time t and its magnitude is given by $E = E_0 \cos \omega t$, where E_0 and ω are constants. Find the position of the particle as a function of time.

Hint: A charge q in a field $\boldsymbol{E}$ experiences a force $q\boldsymbol{E}$.

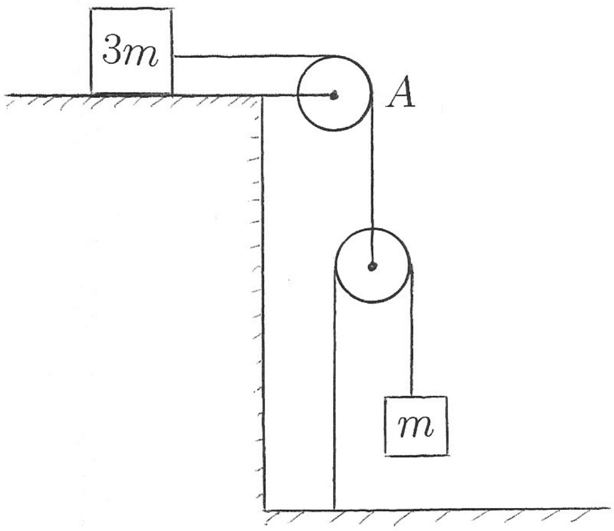
Question 2 [10 marks]

A soldier in a helicopter hovering at a height of 200 m fires his carbine at a ground target directly below him. The bullet (mass m) is discharged at a speed of 400 $\mathrm{m\,s^{-1}}$. It experiences atmospheric drag proportional to the *square* of its speed with a proportionality constant βm, where $\beta = 0.001\ \mathrm{m^{-1}}$. Integrate Newton's second law of motion *once* in order to find the relationship between the speed of the bullet and its position and then use this relationship to calculate the speed at which the bullet hits the target.

Hint: Use the chain rule in the form $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}v}{\mathrm{d}r}\dfrac{\mathrm{d}r}{\mathrm{d}t} = \dfrac{\mathrm{d}v}{\mathrm{d}r}\,v.$

Question 3 [10 marks]

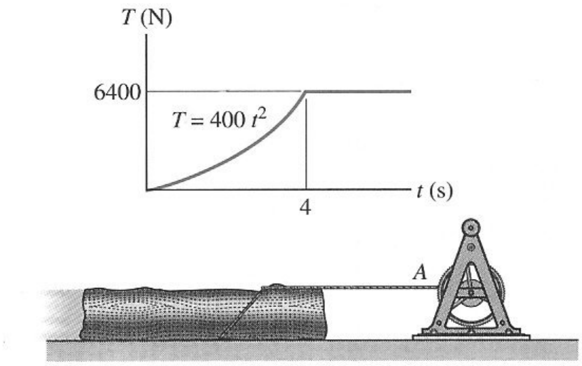
In the figure the horizontal plane is frictionless, the masses of the pulleys and the cords are negligible and the axis of A is fixed so that it cannot translate. Calculate the acceleration of both particles (masses m and $3m$ respectively) as multiples of g.



Question 4 [8 marks]

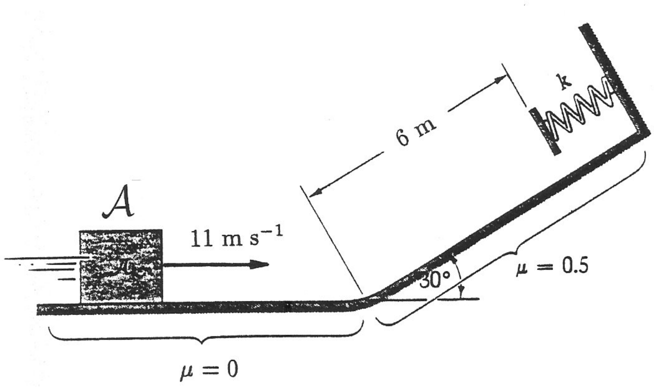
The log in the figure has a mass of 500 kg and rests on the ground for which the friction coefficient is $\mu = 0.4$. The winch delivers a towing force $\boldsymbol{T}$ to its cable at A which varies with time as shown in the graph. Use the Principle of Impulse and Momentum to determine the speed of the log when $t = 5$ s (use $g = 9.81\ \mathrm{m\,s^{-2}}$ in your calculations).

Hint: First determine the force needed to begin moving the log.



Question 5 [12 marks]

Block $\mathcal{A}$, mass 7 kg, translates along a smooth horizontal plane with a speed of 11 $\text{m}\,\text{s}^{-1}$. The friction coefficient between $\mathcal{A}$ and the inclined surface is $\mu = 0.5$, and the spring constant is 450 Nm^{-1}. Use the Principle of Work and Energy to determine the distance that $\mathcal{A}$ moves up the incline (angle with the horizontal is 30°) before coming to rest.



USEFUL INFORMATION

Gravitational acceleration at the earth's surface:

$$g = 9.81 \text{ ms}^{-2}.$$

Well-known integral:

$$\int \frac{u \, \mathrm{d}u}{a + bu^2} = \frac{1}{2b} \ln \left| a + bu^2 \right| .$$

Principle of Impulse and Momentum:

$$\boldsymbol{I} = \int \boldsymbol{F} \, \mathrm{d}t = \Delta \boldsymbol{p}.$$

Principle of Energy and Momentum:

$$\Delta T = W = \int_\Gamma \mathbf{F} \cdot \mathrm{d}\boldsymbol{r}.$$

ROUGH WORK

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