APPLIED MATHEMATICS 1B
APM01B1/APM1B10

Introduction to Dynamics

EXAMINATION

21 November 2019

Duration: 2 hours   **Marks:** 50

Assessor: Prof CM Villet
           Mr KD Anderson
Moderator: Dr GJ Kemp

Student number:

Initials & surname:

Instructions:
1. Answer all the questions.
2. All calculations must be shown.
3. Pocket calculators are permitted, work to at least three significant figures throughout.
4. All quantities are given in SI units.
5. All symbols have their usual meaning and all angles are measured in radians (unless stated otherwise).
6. Vectors are indicated throughout by bold face type, for example $\boldsymbol{a}$, $\boldsymbol{r}$.
7. Useful results are supplied at the end of this paper and can be considered as given.

Question	Marks	Score
1	10	
2	10	
3	10	
4	7	
5	5	
6	8	
Total:	50	

Question 1   [10 marks]

Two cars start from rest at the same location and the same time instant and race along a straight track. Car A undergoes a constant acceleration of 4 m s^{-2} up to a speed of 40 m s^{-1} and then runs at a constant speed. Car B undergoes a constant acceleration of 3 m s^{-2} up to a speed of 45 m s^{-1} and then runs at a constant speed. Which car will win the 5 km race?

Question 2   **[10 marks]**

A projectile is launched vertically from the surface of the earth at a speed of $\lambda\sqrt{gR}$, where R is the radius of the earth, g is the gravitational acceleration constant at the earth's surface and λ is a constant which can be large.

(a) Ignore atmospheric resistance and integrate Newton's second law of motion once in order to find the maximum height reached by the projectile in terms of R and λ.   (9)

    Hint: Use the chain rule in the form $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}v}{\mathrm{d}r}\dfrac{\mathrm{d}r}{\mathrm{d}t} = \dfrac{\mathrm{d}v}{\mathrm{d}r}\,v$.

(b) Discuss the special case $\lambda = \sqrt{2}$.   (1)

Question 3　[10 marks]

The blocks in the figure are in contact as they slide down the inclined plane. The masses of A and B are m and $3m$ respectively, and the friction coefficients between the blocks and the plane are $\frac{2}{3}$ for A and $\frac{1}{3}$ for B. Calculate the common acceleration of the blocks (as a multiple of g) as well as the mutual force between them (as a multiple of mg).
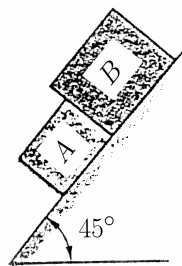
Question 4   [7 marks]

Establish whether the force field $\boldsymbol{F} = yz\,\hat{x} + xz\,\hat{y} + xy\,\hat{z}$ is conservative.

Question 5   [5 marks]

A particle of mass m is at a distance r from the centre of the earth, where r can be significantly larger than the radius of the earth. Assume that gravity is conservative and derive an expression for the potential energy of the particle due to the earth's gravitational field.

Hint: Use a point infinitely far from the earth as a reference point.

Question 6 [8 marks]

The mass in the figure slides from rest at A along the frictionless hemispherical surface. Make use of the conservation of mechanical energy, amongst others, to establish where it will leave the surface.
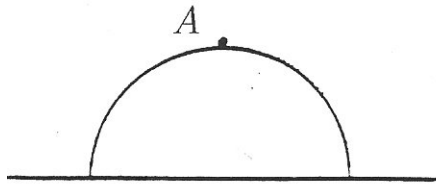
USEFUL INFORMATION

If a body in straight line motion has constant acceleration a for a time t over a distance s and in the process its speed increases from v_0 to v, one has that

$$v = v_0 + at; \qquad s = v_0 t + \frac{1}{2} at^2.$$

Newton's universal law of gravitation:

$$F = \frac{GMm}{r^2}.$$

For the earth's gravitational field we have that

$$GM = gR^2.$$

For a conservative force-field we have that

$$\boldsymbol{F} = -\boldsymbol{\nabla} V; \qquad V(\boldsymbol{r}) = -\int_{\boldsymbol{r_0}}^{\boldsymbol{r}} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r}; \qquad \boldsymbol{\nabla} \times \boldsymbol{F} = \boldsymbol{0}.$$

Conservation of Mechanical Energy:

$$T + V = \text{constant}.$$

Centripetal acceleration:

$$\frac{v^2}{R}.$$

ROUGH WORK

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