UNIVERSITY OF JOHANNESBURG

PROGRAM	:	BACCALAUREUS INGENERIAE *MECHANICAL ENGINEERING*
SUBJECT	:	**Design (Mechanical) 2A**
CODE	:	**OWMMCA2 / OWM2A**
DATE	:	WINTER EXAMINATION - June 2019
DURATION	:	3 hours
WEIGHT	:	50 : 50
TOTAL MARKS	:	100

EXAMINER	:	Dr BW Botha
MODERATOR	:	Dr A Maneschijn
NUMBER OF PAGES	:	4 PAGES AND 1 ANNEXURE

INSTRUCTIONS	:	QUESTION PAPERS MUST BE HANDED IN.
REQUIREMENTS	:	ANSWER BOOKLET.

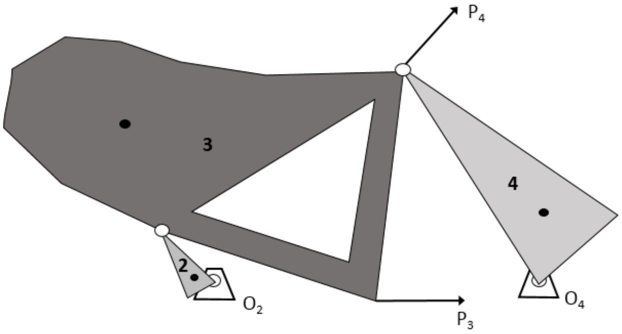
INSTRUCTIONS TO CANDIDATES:

PLEASE ANSWER ALL THE QUESTIONS.

QUESTION 1

40 Marks

A mechanism system consists of four links as indicated in the figure. The parameters for the relevant parts are given in the table. Calculate the relevant information required and complete the relevant matrices with all information required in order to calculate the forces acting on the links given in the simplest form in.



Lengths in mm, angles in degrees, angular acceleration in rad/s^2

link 2	link 3	link 4	link 1	θ_2	θ_3	θ_4	α_2	α_3	α_4
50.8	203.2	177.8	228.6	135	34.02	122.71	25	71.54	–14.19

Angular velocity in rad/sec, mass in kg, mass moment of inertia in kg-m^2, torque in N-m

ω_2	ω_3	ω_4	m_2	m_3	m_4	I_2	I_3	I_4	T_3	T_4
20	1.06	5.61	0.18	7.01	15.76	0.034	0.090	0.034	2.82	4.52

Lengths in mm angles in degrees, linear accelerations in m/s^2

R_{g_2} mag	R_{g_2} ang	R_{g_3} mag	R_{g_3} ang	R_{g_4} mag	R_{g_4} ang	a_{g_2} mag	a_{g_2} ang	a_{g_3} mag	a_{g_3} ang
12.7	30	76.2	75	50.8	–40	5.09	341.42	19.05	295.98

Linear accelerations in m/s^2, forces in N, lengths in mm, angles in degrees

a_{g_4} mag	a_{g_4} ang	F_{P_3} mag	δF_{P_3} ang	R_{P_3} mag	δR_{P_3} ang	F_{P_4} mag	δF_{P_4} ang	R_{P_4} mag	δR_{P_4} ang
1.75	286.97	40.03	0	152.4	–60	71.17	60	177.8	0

QUESTION 2

20 Marks

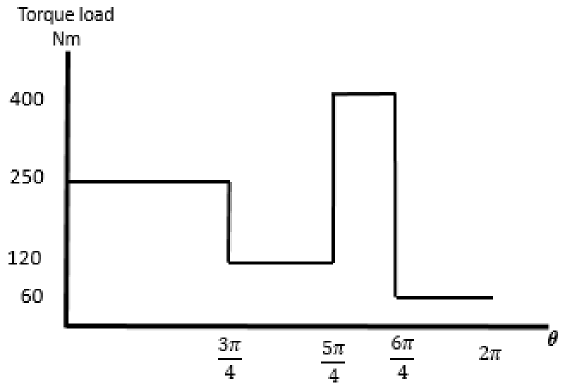
A bearing set is to be used in a mining application where it will be exposed to primarily radial loads. Failure of previous designs indicated a limited axial load to be generated under certain conditions. In order to accommodate the axial load the original bearing is to be replaced by an angular contact bearing. The application requires a design life of 40 000 hours at a speed of 450 rpm. The load to be carried by the bearing set is 6750N and is equally distributed between the bearings. The application is exposed to a light shock represented by an application factor of 1.2. The reliability goal for the bearing set is given to be 96%. Determine the following:

a. Multiple of rating life required (x_D)
b. Catalogue rating (C_{10})
c. Specify a suitable bearing
d. The overall reliability expected of the final design configuration.

QUESTION 3

20 Marks

A manufacturing process has a base torque load requirement of 60 Nm. When in operation the process requires 250 Nm from zero to $3\pi/4$, 120 Nm from $3\pi/4$ to $5\pi/4$ and 400 Nm from $5\pi/4$ to $7\pi/4$ before dropping to base torque during each revolution of the shaft. The process requires that the shaft speed does not vary by more than 3% from the average speed of 980 rpm. The flywheel and the shaft are both made of RQC-100 Steel. In order to limit weight it is decided to follow a rim design with limitation on OD of 280 mm and ID of 200 mm mounted on a spoke assembly.



Neglecting the weight of the spokes, calculate the:

a. Required width of the flywheel rim
b. Minimum and maximum speed of the shaft.
c. The circumferential hoop and radial stresses experienced by the flywheel if it is assumed that the flywheel is fitted to the spokes without generating an internal pressure.

Show all the steps and the energy balance in your calculations and justify your steps, wherever necessary.

Assume properties of RQC-100 Steel to be as follows: Density: 7860 kg/m^3, Yield Strength: 683 MPa, Ultimate Tensile Strength: 758 MPa, Poisson's Ratio: 0.3, Modulus of elasticity: 207 GPa

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QUESTION 4

20 Marks

a) Indicate a suitable type of fit for the following applications where:
   i) Parts need to move freely, but with good accuracy at moderate speeds
   ii) Stationary parts need to be aligned with a snug fit, but must allow free assembly and disassembly
   iii) Relative movement between parts are to be prevented, but maintenance will require removal and replacement of the parts on a more regular basis
   iv) Parts are to be located accurately before being secured
   v) Parts are to be permanently joined without potential for relative movement or general maintenance

b) A 30 mm diameter shaft manufactured from tool steel is supplied with an 8 mm keyway. The shaft needs to transmit 25 kW of power at a speed or 1750 rpm. The shaft will be exposed to a light shock load with an application factor of 1.2. The customer requires a minimum factor of safety of 2. As engineer you are tasked to determine the required length of the key using standard key stock with a yield strength of 450 MPa to allow the transmission of the power without the key failing.

ANNEXURE

Brakes

Long shoe

$$dN = \frac{p_{\max} br \sin\theta \, d\theta}{\sin\theta_a} \qquad F = \frac{M_N - M_f}{c} \qquad F = \frac{M_N + M_f}{c}$$

$$M_N = \frac{abr\,p_{\max}}{4\sin\theta_a}\left[2(\theta_2-\theta_1) - \sin 2\theta_2 + \sin 2\theta_1\right]$$

$$M_F = \frac{\mu br\,p_{\max}}{\sin\theta_a}\left[-r\left(\cos\theta_2 - \cos\theta_1\right) - \frac{a}{2}\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right]$$

$$T = \frac{\mu p_{\max} br^2}{\sin\theta_a}\left(\cos\theta_1 - \cos\theta_2\right)$$

$$R_x = -F_x + \frac{p_{\max} br}{4\sin\theta_a}\left\{2\left(\sin^2\theta_2 - \sin^2\theta_1\right) - \mu\left[2(\theta_2-\theta_1) - \sin 2\theta_2 + \sin 2\theta_1\right]\right\}$$

$$R_y = -F_y + \frac{p_{\max} br}{4\sin\theta_a}\left\{2(\theta_2-\theta_1) - \sin 2\theta_2 + \sin 2\theta_1 + 2\mu\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right\}$$

$$R_x = -F_x + \frac{p_{\max} br}{4\sin\theta_a}\left\{2\left(\sin^2\theta_2 - \sin^2\theta_1\right) + \mu\left[2(\theta_2-\theta_1) - \sin 2\theta_2 + \sin 2\theta_1\right]\right\}$$

$$R_y = -F_y + \frac{p_{\max} br}{4\sin\theta_a}\left\{2(\theta_2-\theta_1) - \sin 2\theta_2 + \sin 2\theta_1 - 2\mu\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right\}$$

Energy

$$\dot\theta = \dot\theta_1 - \dot\theta_2 = \omega_1 - \omega_2 - T\left(\frac{I_1 + I_2}{I_1 I_2}\right)t$$

$$t_1 = \frac{I_1 I_2\left(\omega_1 - \omega_2\right)}{T\left(I_1 + I_2\right)}$$

$$u = T\dot\theta = T\left[\omega_1 - \omega_2 - T\left(\frac{I_1 + I_2}{I_1 I_2}\right)t\right]$$

$$E = \frac{I_1 I_2\left(\omega_1 - \omega_2\right)^2}{2\left(I_1 + I_2\right)} \qquad \Delta T = \frac{E}{WC_p}$$

$$\frac{T - T_\infty}{T_1 - T_\infty} = \exp\left(\frac{-h_{CR}A}{W C_p}\right)$$

$$E = {}^1\!/_2\, I(\omega_1^2 - \omega_2^2)$$

$$Wear = K_A A p_1 V t$$

Band Brakes

$$\frac{F_1}{F_2} = e^{\mu\phi}$$

$$H_{loss} = h_{CR} A\left(T - T_\infty\right) = \left(h_r + f_v h_c\right) A\left(T - T_\infty\right)$$

$$T_{\max} = T_\infty + \frac{\Delta T}{1 - \exp\left(\beta t_1\right)} \qquad with \quad \beta = \left(\frac{-h_{CR}A}{W C_p}\right)$$

Short shoe

$$d_4 W + \mu P d_1 - d_3 P = 0$$

$$P = \frac{d_4 W}{d_3 - \mu d_1}$$

$$T_C = F_{friction}\, r = \mu r P = \frac{\mu r d_4 W}{d_3 - \mu d_1}$$

$$d_4 W - \mu P d_2 - d_3 P = 0$$

$$P = \frac{d_4 W}{d_3 + \mu d_2}$$

$$T_D = F r = \mu r P = \frac{\mu r d_4 W}{d_3 + \mu d_2}$$

Disk Brakes

$$F = p_{\max} r_i\left(\theta_2 - \theta_1\right)\left(r_{outer} - r_{inner}\right) \qquad F = p_{\max}\left(\theta_2 - \theta_1\right)\tfrac{1}{2}\left(r_o^2 - r_i^2\right)$$

$$T = \tfrac{1}{2}\mu p_{\max} r_i\left(\theta_2 - \theta_1\right)\left(r_o^2 - r_i^2\right) \qquad T = \mu p_{\max}\left(\theta_2 - \theta_1\right)\tfrac{1}{3}\left(r_o^3 - r_i^3\right)$$

$$r_e = \frac{\left(r_o + r_i\right)}{2} \qquad r_e = \frac{2}{3}\frac{\left(r_o^3 - r_i^3\right)}{\left(r_o^2 - r_i^2\right)}$$

Bearings

$$C_{10} = F_R = F_D \frac{(\mathcal{L}_D n_D 60)^{1/a}}{(\mathcal{L}_R n_R 60)^{1/a}} \qquad C_{10} = a_f F_D\left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}}\right]^{1/a}$$

$$x_D = \frac{L_D}{L_R} = \frac{\mathcal{L}_D n_D 60}{L_{10}} \qquad F_R(\mathcal{L}_R n_R 60)^{1/a} = F_D(\mathcal{L}_D n_D 60)^{1/a}$$

$$Re = 1 - \left(\frac{x_D\left(\frac{a_f F_D}{C_{10}}\right)^a - x_0}{\theta - x_0}\right)^b$$

Manufacturer	Rating Life, revolutions	Weibull Parameters Rating Lives x_0	θ	b
1	90(10^6)	0	4.48	1.5
2	1(10^6)	0.02	4.459	1.483

Table 16–5

Friction Materials for Clutches and brakes

Material	Friction Coefficient Wet	Friction Coefficient Dry	Max. Temperature °F	Max. Temperature °C	Max. Pressure psi	Max. Pressure kPa
Cast iron on cast iron	0.05	0.15–0.20	600	320	150–250	1000–1750
Powdered metal* on cast iron	0.05–0.1	0.1–0.4	1000	540	150	1000
Powdered metal* on hard steel	0.05–0.1	0.1–0.3	1000	540	300	2100
Wood on steel or cast iron	0.16	0.2–0.35	300	150	60–90	400–620
Leather on steel or cast iron	0.12	0.3–0.5	200	100	10–40	70–280
Cork on steel or cast iron	0.15–0.25	0.3–0.5	200	100	8–14	50–100
Felt on steel or cast iron	0.18	0.22	280	140	5–10	35–70
Woven asbestos* on steel or cast iron	0.1–0.2	0.3–0.6	350–500	175–260	50–100	350–700
Molded asbestos* on steel or cast iron	0.08–0.12	0.2–0.5	500	260	50–150	350–1000
Impregnated asbestos* on steel or cast iron	0.12	0.32	500–750	260–400	150	1000
Carbon graphite on steel	0.05–0.1	0.25	700–1000	370–540	300	2100

*The friction coefficient can be maintained with ±5 percent for specific materials in this group.

Table 11–2

Dimensions and Load Ratings for Single-Row 02-Series Deep-Groove and Angular-Contact Ball Bearings

Bore, mm	OD, mm	Width, mm	Fillet Radius, mm	Shoulder Diameter, mm d_S	Shoulder Diameter, mm d_H	Load Ratings, kN Deep Groove C_{10}	Deep Groove C_0	Angular Contact C_{10}	Angular Contact C_0
10	30	9	0.6	12.5	27	5.07	2.24	4.94	2.12
12	32	10	0.6	14.5	28	6.89	3.10	7.02	3.05
15	35	11	0.6	17.5	31	7.80	3.55	8.06	3.65
17	40	12	0.6	19.5	34	9.56	4.50	9.95	4.75
20	47	14	1.0	25	41	12.7	6.20	13.3	6.55
25	52	15	1.0	30	47	14.0	6.95	14.8	7.65
30	62	16	1.0	35	55	19.5	10.0	20.3	11.0
35	72	17	1.0	41	65	25.5	13.7	27.0	15.0
40	80	18	1.0	46	72	30.7	16.6	31.9	18.6
45	85	19	1.0	52	77	33.2	18.6	35.8	21.2
50	90	20	1.0	56	82	35.1	19.6	37.7	22.8
55	100	21	1.5	63	90	43.6	25.0	46.2	28.5
60	110	22	1.5	70	99	47.5	28.0	55.9	35.5
65	120	23	1.5	74	109	55.9	34.0	63.7	41.5
70	125	24	1.5	79	114	61.8	37.5	68.9	45.5
75	130	25	1.5	86	119	66.3	40.5	71.5	49.0
80	140	26	2.0	93	127	70.2	45.0	80.6	55.0
85	150	28	2.0	99	136	83.2	53.0	90.4	63.0
90	160	30	2.0	104	146	95.6	62.0	106	73.5
95	170	32	2.0	110	156	108	69.5	121	85.0

Table 11–3

Dimensions and Basic Load Ratings for Cylindrical Roller Bearings

	02-Series				03-Series			
Bore, mm	OD, mm	Width, mm	Load Rating, kN C_{10}	C_0	OD, mm	Width, mm	Load Rating, kN C_{10}	C_0
25	52	15	16.8	8.8	62	17	28.6	15.0
30	62	16	22.4	12.0	72	19	36.9	20.0
35	72	17	31.9	17.6	80	21	44.6	27.1
40	80	18	41.8	24.0	90	23	56.1	32.5
45	85	19	44.0	25.5	100	25	72.1	45.4
50	90	20	45.7	27.5	110	27	88.0	52.0
55	100	21	56.1	34.0	120	29	102	67.2
60	110	22	64.4	43.1	130	31	123	76.5
65	120	23	76.5	51.2	140	33	138	85.0
70	125	24	79.2	51.2	150	35	151	102
75	130	25	93.1	63.2	160	37	183	125
80	140	26	106	69.4	170	39	190	125
85	150	28	119	78.3	180	41	212	149
90	160	30	142	100	190	43	242	160
95	170	32	165	112	200	45	264	189
100	180	34	183	125	215	47	303	220
110	200	38	229	167	240	50	391	304
120	215	40	260	183	260	55	457	340
130	230	40	270	193	280	58	539	408
140	250	42	319	240	300	62	682	454

Flywheels

$$\int_{\theta_{min}}^{\theta_{max}} (T_l - T_{avg})\, d\theta = \int_{\omega_{min}}^{\omega_{max}} I_m\, \omega d\omega = \frac{I_m}{2}(\omega_{max}^2 - \omega_{min}^2) = K_E$$

$$C_s = \frac{\omega_{max} - \omega_{min}}{\omega_{ave}} = 2\frac{\omega_{max} - \omega_{min}}{\omega_{max} + \omega_{min}} \qquad I_m = \frac{K_E}{C_s \omega_{ave}^2}$$

$$I_m = \frac{m d_i^2}{8} = \rho \frac{\pi d^2}{4} t \frac{d^2}{8}$$ (Solid circular cross-section)

$$I_m = \frac{m(d_o^2 + d_i^2)}{8} = \rho \frac{\pi(d_o^2 - d_i^2)}{4} t \frac{(d_o^2 + d_i^2)}{8}$$ (Hollow circular cross-section)

$$\sigma_\theta = \frac{3+\nu}{8} \cdot \rho_{steel} \cdot w_{ave}^2 \cdot \left(r_i^2 + r_o^2 + \frac{r_i^2 \cdot r_o^2}{r^2} - \frac{(1+3\nu)}{3+\nu} \cdot r^2 \right) + \frac{p_i \cdot r_i^2 \cdot \left(1 + \frac{r_o^2}{r^2}\right)}{r_o^2 - r_i^2}$$

$$\sigma_r = \frac{3+\nu}{8} \cdot \rho_{steel} \cdot w_{ave}^2 \cdot \left(r_i^2 + r_o^2 - \frac{r_i^2 \cdot r_o^2}{r^2} - r^2 \right) + \frac{p_i \cdot r_i^2 \cdot \left(1 - \frac{r_o^2}{r^2}\right)}{r_o^2 - r_i^2}$$

Circumferential strain: $\varepsilon_\theta = \frac{\sigma_\theta}{E} - \frac{\nu \sigma_r}{E}$ Deflection: $\delta = \varepsilon_\theta r$

Stress due to interference: $p_i = \frac{E\delta(r_o^2 - r_i^2)}{2 r_i r_o^2}$

Flywheels:

$$\int_{\theta_{\omega_{min}}}^{\theta_{\omega_{max}}} \left(T_l - T_{avg}\right) d\theta = \int_{\omega_{min}}^{\omega_{max}} I_m \omega \, d\omega = \frac{I_m}{2}\left(\omega_{max}^2 - \omega_{min}^2\right) = K_e$$

$$C_f = \frac{\omega_{max} - \omega_{min}}{\omega_{avg}} = 2\frac{\omega_{max} - \omega_{min}}{\omega_{max} + \omega_{min}} \qquad I_m = \frac{K_e}{C_f \omega_{avg}^2}$$

$$I_m = \frac{md^2}{8} = \rho \frac{\pi d^2}{4} t \frac{d^2}{8}$$ (Solid circular cross-section)

$$I_m = \frac{m\left(d_o^2 + d_i^2\right)}{8} = \rho \frac{\pi\left(d_o^2 - d_i^2\right)}{4} t \frac{\left(d_o^2 + d_i^2\right)}{8}$$ (Hollow cross-section)

$$\sigma_\theta = \sigma_{\theta\omega} + \sigma_{\theta p} \quad , \quad \sigma_r = \sigma_{r\omega} + \sigma_{rp}$$

$$\sigma_\theta = \frac{3+\nu}{8}\rho\omega^2\left[r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{(1+3\nu)}{3+\nu} r^2\right] + \frac{p_i r_i^2\left(1 + \frac{r_o^2}{r^2}\right)}{r_o^2 - r_i^2}$$

$$\sigma_r = \frac{3+\nu}{8}\rho\omega^2\left[r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right] + \frac{p_i r_i^2\left(1 - \frac{r_o^2}{r^2}\right)}{r_o^2 - r_i^2}$$

Brittle fracture: $N = \dfrac{S_{ut}}{\sigma_1}$

Yield Failure: $N = \dfrac{S_y}{\sigma_v}$

$$\sigma_v = \sqrt{\sigma_\theta^2 + \sigma_r^2 - \sigma_\theta \sigma_r}$$

Circumferential strain: $\varepsilon_\theta = \dfrac{\sigma_\theta}{E} - \dfrac{\nu\sigma_r}{E}$

Deflection: $\delta = \varepsilon_\theta r$

Stress due to interference : $p_i = \dfrac{E\delta_i\left(r_o^2 - r_i^2\right)}{2 r_i r_o^2}$

MECHANISMS
DOF

Computing DOF - Spatial: $M = 6(L-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - J_5$

M – Mobility/DOF
L – No. of links
J_1 – No. of joints capturing 5DOF
J_2 – No. of joints capturing 4DOF
J_3 – No. of joints capturing 3DOF
J_4 – No. of joints capturing 2DOF
J_5 – No. of joints capturing 1DOF

Computing DOF - Planar:

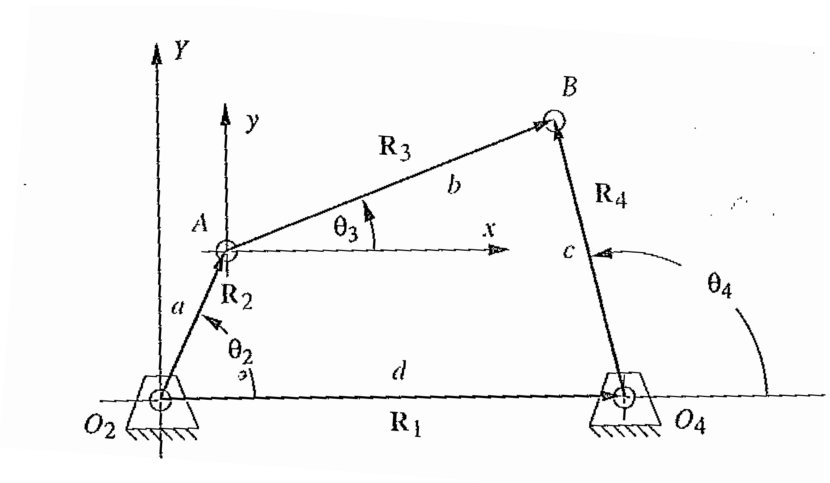
$$M = 3L - 2J - 3G = 3(L-1) - 2J = 3(L-1) - 2J_1 - J_2$$

M – DOF/Mobility, G –Ground link, L – No. of links,
J_1 – No. of full joints, J_2 – No. of half joints

Coordinate Transformation

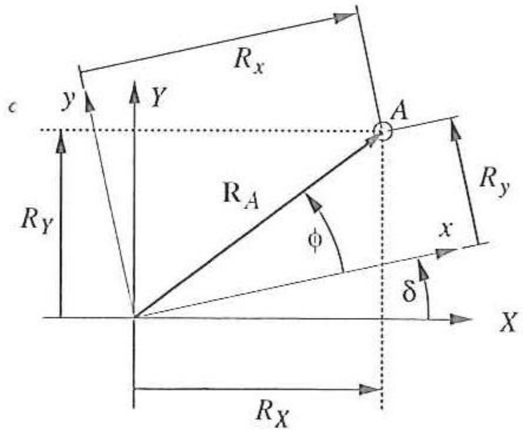
$$\begin{bmatrix} R_X \\ R_Y \end{bmatrix} = \begin{bmatrix} Cos\delta & -Sin\delta \\ Sin\delta & Cos\delta \end{bmatrix} \begin{bmatrix} R_x \\ R_y \end{bmatrix}$$

Four Bar Linkage



Position Analysis:

$$\vec{R_2} + \vec{R_3} - \vec{R_4} - \vec{R_1} = 0 \quad , \quad ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j0} = 0$$



$$k_1 = \frac{d}{a} \quad , \quad k_2 = \frac{d}{c} \quad , \quad k_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac} \quad ,$$

$$k_4 = \frac{d}{b} \quad , \quad k_5 = \frac{c^2 - d^2 - a^2 - b^2}{2ab}$$

$$A = Cos\theta_2 - k_1 + k_3 - k_2 Cos\theta_2 \quad , \quad B = -2Sin\theta_2 \quad , \quad C = k_1 - k_2 Cos\theta_2 + k_3 - Cos\theta_2$$

$$D = Cos\theta_2 - k_1 + k_5 + k_4 Cos\theta_2 \quad , \quad E = -2Sin\theta_2 \quad , \quad F = k_1 + k_4 Cos\theta_2 + k_5 - Cos\theta_2$$

$$Tan\frac{\theta_4}{2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad \therefore \quad \theta_4 = 2Tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right]$$

$$Tan\frac{\theta_3}{2} = \frac{-E \pm \sqrt{E^2 - 4DF}}{2D} \quad \therefore \quad \theta_3 = 2Tan^{-1}\left[\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right]$$

Velocity Analysis:

$$\vec{V}_A + \vec{V}_{BA} - \vec{V}_B = 0 \qquad a\,j\omega_2 e^{j\theta_2} + b\,j\omega_3 e^{j\theta_3} - c\,j\omega_4 e^{j\theta_4} = 0$$

$$\omega_3 = \frac{a\omega_2}{b}\frac{Sin(\theta_4 - \theta_2)}{Sin(\theta_3 - \theta_4)} \qquad \omega_4 = \frac{a\omega_2}{c}\frac{Sin(\theta_2 - \theta_3)}{Sin(\theta_4 - \theta_3)}$$

$$\vec{V}_A = a\omega_2(-Sin\theta_2 + jCos\theta_2)\ ,\ \vec{V}_B = c\omega_4(-Sin\theta_4 + jCos\theta_4)\ ,\ \vec{V}_{BA} = b\omega_3(-Sin\theta_3 + jCos\theta_3)$$

Acceleration Analysis:

$$\left(j^2 a\omega_2^2 e^{j\theta_2} + ja\alpha_2 e^{j\theta_2}\right) + \left(j^2 b\omega_3^2 e^{j\theta_3} + jb\alpha_3 e^{j\theta_3}\right) - \left(j^2 c\omega_4^2 e^{j\theta_4} + jc\alpha_4 e^{j\theta_4}\right) = 0$$

$$A = cSin\theta_4 \qquad B = bSin\theta_3$$

$$C = a\alpha_2 Sin\theta_2 + a\omega_2^2 Cos\theta_2 + b\omega_3^2 Cos\theta_3 - c\omega_4^2 Cos\theta_4$$

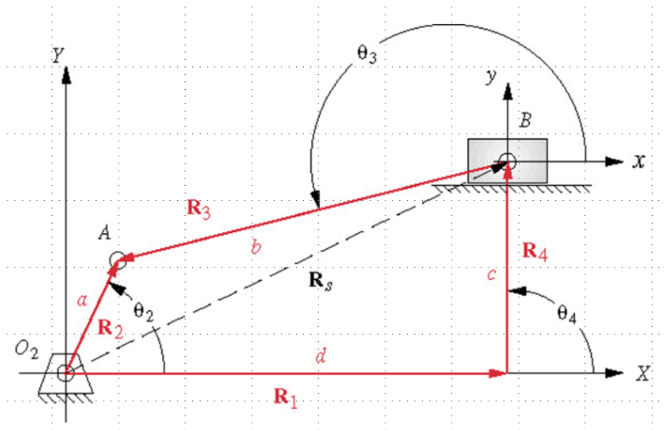
$$D = cCos\theta_4$$

$$E = bCos\theta_3$$

$$F = a\alpha_2 Cos\theta_2 - a\omega_2^2 Sin\theta_2 - b\omega_3^2 Sin\theta_3 + c\omega_4^2 Sin\theta_4$$

$$\alpha_3 = \frac{C.D - A.F}{A.E - B.D} \qquad \alpha_4 = \frac{C.E - B.F}{A.E - B.D}$$

Four Bar Slider Crank



Position Analysis:

$$\vec{R}_2 - \vec{R}_3 - \vec{R}_4 - \vec{R}_1 = 0 \ ,\ ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j0} = 0$$

$$\theta_{31} = \arcsin\left(\frac{a\sin(\theta_2) - c}{b}\right)$$

$$\theta_{32} = \arcsin\left(-\frac{a\sin(\theta_2) - c}{b}\right) + \pi \ ,\quad \theta_{33} = \arcsin\left(-\frac{a\sin(\theta_2) + c}{b}\right) + \pi$$

$$d = a\cos(\theta_2) - b\cos(\theta_3)$$

Velocity Analysis:

$$\omega_3 = \frac{a}{b}\frac{\cos(\theta_2)}{\cos(\theta_3)}\omega_2 \ ,\quad \dot{d} = -a\omega_2\sin(\theta_2) + b\omega_3\sin(\theta_3)$$

$$\vec{V}_A = a\omega_2(-sin\theta_2 + jcos\theta_2)\ ,\ \vec{V}_{AB} = b\omega_3(-sin\theta_3 + jcos\theta_3)\ ,\ \vec{V}_B = \dot{d} = -a\omega_2\, sin\theta_2 + b\omega_3 sin\theta_3$$

Acceleration Analysis:

$$\left(j^2 a\omega_2^2 e^{j\theta_2} + ja\alpha_2 e^{j\theta_2}\right) - \left(j^2 b\omega_3^2 e^{j\theta_3} + jb\alpha_3 e^{j\theta_3}\right) - \ddot{d} = 0$$

$$\alpha_3 = \frac{a\alpha_2\cos(\theta_2) - a\,\omega_2^2\sin(\theta_2) + b\omega_3^2\sin(\theta_3)}{b\cos(\theta_3)}$$

$$\ddot{d} = -a\,\alpha_2\sin(\theta_2) - a\,\omega_2^2\cos(\theta_2) + b\alpha_3\sin(\theta_3) + b\,\omega_3^2\cos(\theta_3)$$

Mechanical Advantage & Efficiency

Mechanical Advantage: $m_A = \frac{F_{out}}{F_{in}} = \frac{\omega_{in}}{\omega_{out}}\frac{r_{in}}{r_{out}} = \left(\frac{a\sin\mu}{c\sin\nu}\right)\left(\frac{r_{in}}{r_{out}}\right)$

For 100% efficiency:

$$P_{out} = P_{in} \Rightarrow \frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}}$$

Efficiency:

$$\varepsilon = \frac{P_{out}}{P_{in}}$$

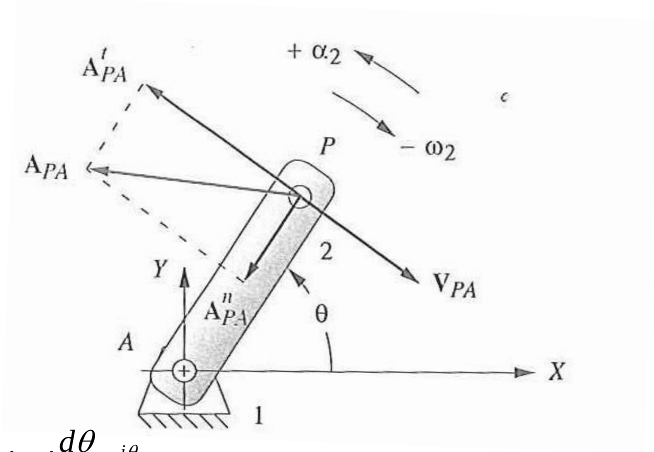
General Formulae

Angular velocity & acceleration/velocity & acceleration at a point:

$$\omega = \frac{d\theta}{dt} \quad , \quad \alpha = \frac{d\omega}{dt} \quad , \quad V = \frac{dR}{dt} \quad , \quad A = \frac{dV}{dt} \quad , \quad v = r\omega$$

A is acceleration vector at a point of interest, *V* is velocity vector at the point of interest and R is the position vector of the point of interest. α is the angular acceleration of a link, *r* is the radius of the link, *ω* is the angular velocity of the rigid link and *v* is the magnitude of velocity.

Expressions for 1-DOF link:

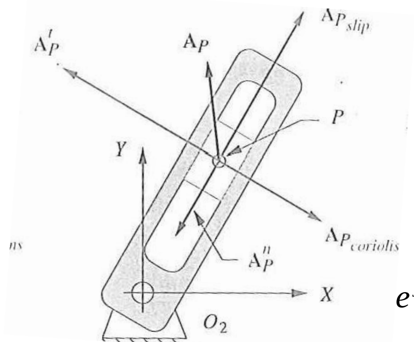


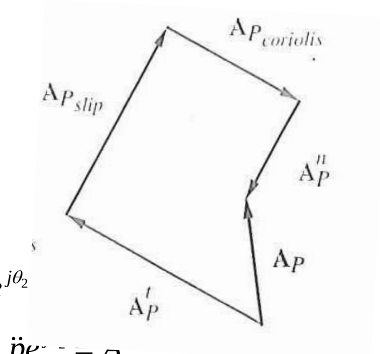
$$R_{PA} = pe^{j\theta}$$

$$V_{PA} = \frac{dR_{PA}}{dt} = pje^{j\theta}\frac{d\theta}{dt} = p\omega je^{j\theta}$$

$$A_{PA} = \frac{dV_{PA}}{dt} = p\,j\frac{d\omega}{dt}e^{j\theta} + p\,j\omega j\frac{d\theta}{dt}e^{j\theta} = p\,j\alpha e^{j\theta} - p\omega^2 e^{j\theta}$$

Coriolis Acceleration:





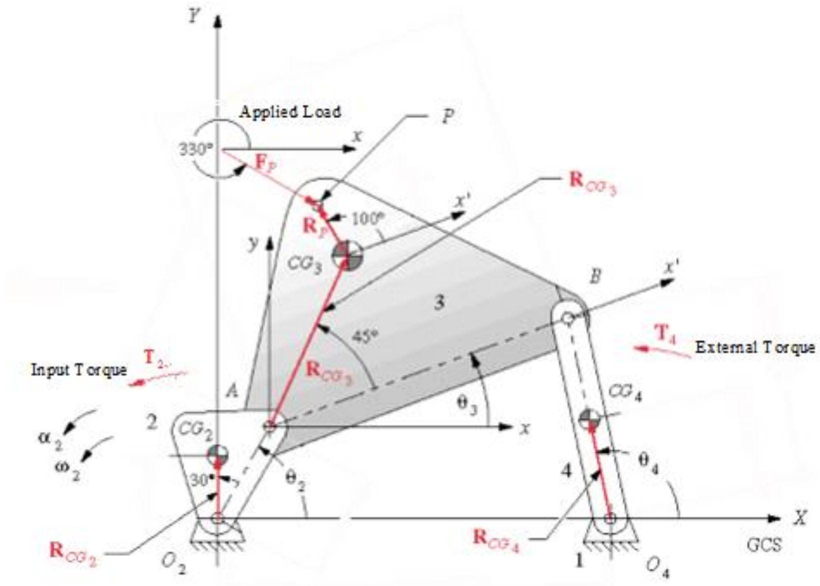
$$\ldots e^{j\theta_2} + \dot{p}e^{j\theta_2}$$

$$A_P = p\alpha_2 je^{j\theta_2} - p\omega_2^2 e^{j\theta_2} + 2\dot{p}j\omega_2 e^{j\theta_2} + \ddot{p}e^{j\theta_2} = A_{P_{tangential}} + A_{P_{normal}} + A_{P_{coriolis}} + A_{P_{slip}}$$

EOM for a Planar System:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum T = I_G\alpha$$

Dynamic Equations for Four-bar linkage:



$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -R_{12y} & R_{12x} & -R_{32y} & R_{32x} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & R_{23y} & -R_{23x} & -R_{43y} & R_{43x} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & \pm\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{32x} \\ F_{32y} \\ F_{43x} \\ F_{43y} \\ F_{14y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G2x} \\ m_2 a_{G2y} \\ I_{G2}\alpha_2 \\ m_3 a_{G3x} \\ m_3 a_{G3y} \\ I_{G3}\alpha_3 \\ m_4 a_{G4x} - F_{P_x} \\ -F_{P_y} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -R_{12y} & R_{12x} & -R_{32y} & R_{32x} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & R_{23y} & -R_{23x} & -R_{43y} & R_{43x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & R_{34y} & -R_{34x} & -R_{14y} & R_{14x} & 0 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{32x} \\ F_{32y} \\ F_{43x} \\ F_{43y} \\ F_{14x} \\ F_{14y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G2x} \\ m_2 a_{G2y} \\ I_{G2}\alpha_2 \\ m_3 a_{G3x} - F_{P3x} \\ m_3 a_{G3y} - F_{P3y} \\ I_{G3}\alpha_3 - T_3 - R_{P3x}F_{P3y} + R_{P3y}F_{P3x} \\ m_4 a_{G4x} - F_{P4x} \\ m_4 a_{G4y} - F_{P4y} \\ I_{G4}\alpha_4 - T_4 - R_{P4x}F_{P4y} + R_{P4y}F_{P4x} \end{bmatrix}$$

Four Bar Slider Crank