UNIVERSITY OF JOHANNESBURG

PROGRAM : B ENG TECH
PHYSICAL & EXTRACTION METALLURGY

SUBJECT : **HEAT & MASS TRANSFER II**

CODE : **HMTMTA2**

DATE : SSA EXAMINATION
18 JULY 2019

DURATION : SESSION 1 08:00 - 11:00

WEIGHT : 40 : 60

TOTAL MARKS : 90

EXAMINER : MR GA COMBRINK

MODERATOR : MR J PROZZI

NUMBER OF PAGES : 4 PAGES AND 6 ANNEXURES
TOTAL 10 PAGES

INSTRUCTIONS : ALL THE ANSWERS MUST BE COMPLETED IN THE EXAM SCRIPS AND HANDED IN
QUESTION PAPERS MUST BE HANDED IN.

REQUIREMENTS : 1 POCKET CALCULATOR
NO CORRECTION FLUID SHALL BE USED
ALL WORK SHALL BE HANDED IN.

INSTRUCTIONS TO CANDIDATES:

PLEASE ANSWER ALL THE QUESTIONS.

REFER TO APPENDICES FOR FURTHER INFORMATION AND EQUATIONS THAT MAY BE REQUIRED IN ANSWERING THE QUESTION IN EACH CASE.

QUESTION 1

Heat Removal from Semi-Infinite Solid

A large flat Copper plate 80mm thick has been heated until it is at a constant and homogeneous temperature of 180ºC. The surface (on both sides) is suddenly cooled to 85°C. What is the total heat removed from the slab per unit surface area when the temperature at a depth 2 mm has dropped to 120°C?

$\alpha = 8.4 \times 10^{-5}\ m^2/s \quad k = 400 W/m.°C$

(See Appendix B Sheet for equations, and further data. Also refer to attached TableA-1 at Appendix A for relevant erf function values) **[12]**

QUESTION 2

Water at 37°C flows through a tube of diameter 30mm at 1litre/second, is the flow turbulent? (Show all your calculations.) $Re_d = U_m d/$ or $Re_x = U_\infty x/v$

To Estimate the Dynamic viscosity [cP] of Water use the following equation

$$\mu = 0.0168 \times \rho \times T^{-0.88}$$

where μ = Viscosity [cP]

ρ = Density [Kg/m^3]

T = Temperature [°C]

and

$$\nu = \frac{\mu}{\rho}$$

ν = ***Kinematic viscosity [cSt]***

Water density at various temperatures	
Temperature (°C)	Density (kg/m^3)
40	992.2000
30	995.6502
25	997.0479
22	997.7735
20	998.2071
15	999.1026
10	999.7026
4	999.9720
0	999.8395

[15]

QUESTION 3

Argon gas at 78ºC and 5 atmosphere pressure flows over a flat steel plate at a speed of 2 m/s. Calculate the boundary layer thickness at distances of 350 mm from the leading edge of the plate. Assume that the mass flow that enters the boundary layer between x = 35cm is 5.232 x 10^{-3} kg/s. The viscosity of gas at 78ºC is 0.0002099 Poise. Assume unit depth in the z direction. Also assume that the plate is heated over its entire length to a temperature of 252ºC. Also calculate the heat transferred in the first 350mm of the plate.

See Equations and data for helpful information in Appendices

[17]

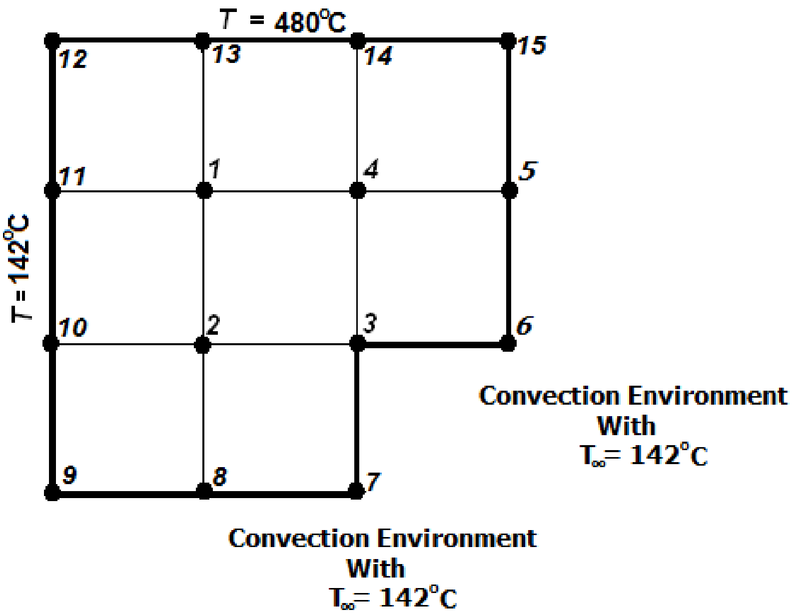
QUESTION 4

What is the difference between laminar flow and turbulent flow velocity profiles? Sketch the velocity profile for turbulent flow showing the differences mentioned above. Explain the physical difference between laminar and turbulent flow

[12]

QUESTION 5

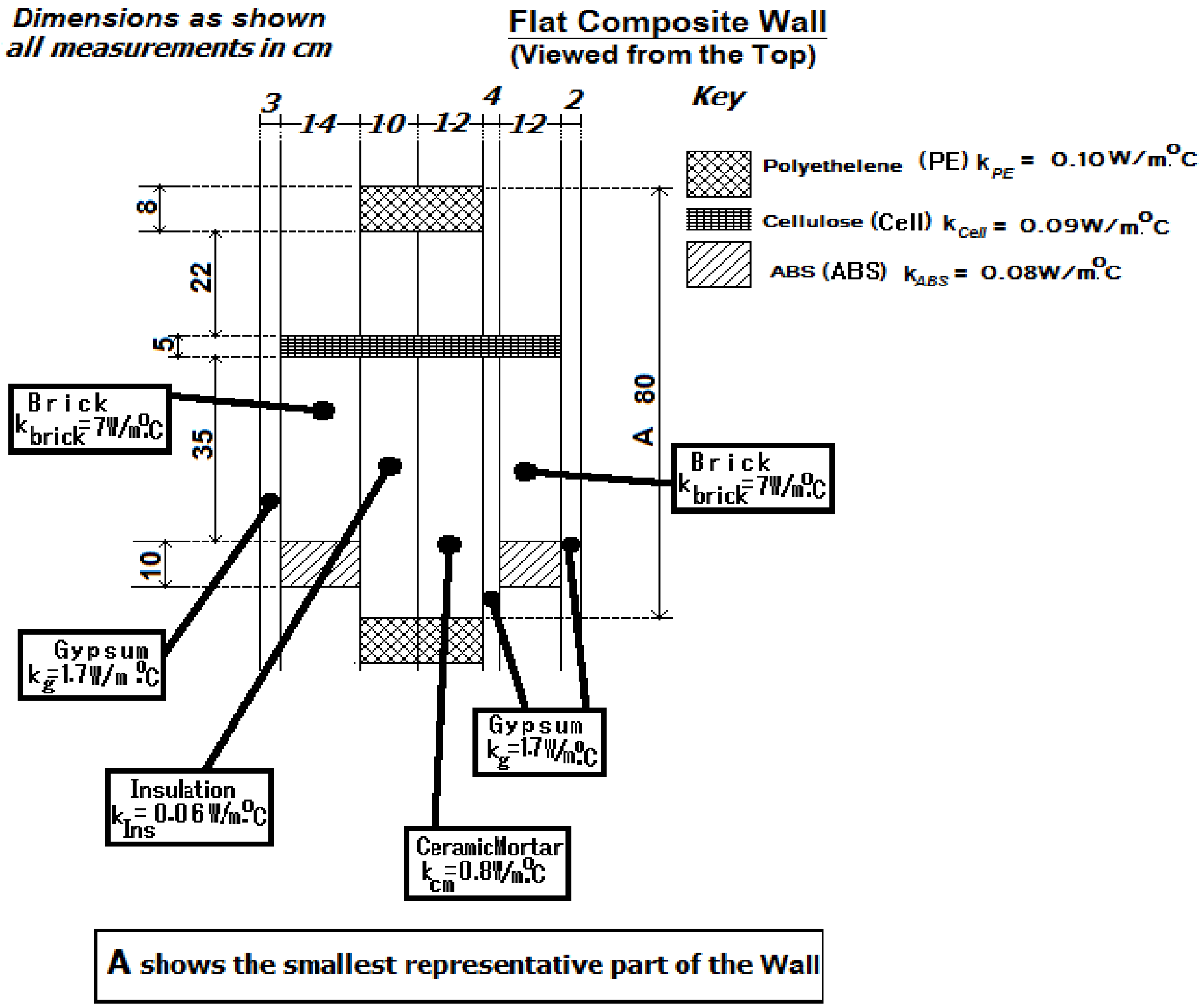
Give equations that mathematically enable one to calculate the temperatures at each of the numbered points 1 to 15 (exclude node 12) in the accompanying sketch if a piece of material's surface is at the temperatures and subjected to the convection environment given on the sketch below. Clearly set out any assumptions that you make. h = 544W/m^2.℃
k = 272W/m.℃ and $\Delta y = \Delta x$ = 0.5m



[14]

QUESTION 6

Draw the equivalent resistance circuit for the wall in the sketch below and calculate equivalent overall thermal resistance in the system: -



[20]

Total Marks **[90]**

Appendix A "erf" Function values

Table The error function.

$\frac{x}{2\sqrt{\alpha\tau}}$	$\text{erf}\,\frac{x}{2\sqrt{\alpha\tau}}$	$\frac{x}{2\sqrt{\alpha\tau}}$	$\text{erf}\,\frac{x}{2\sqrt{\alpha\tau}}$	$\frac{x}{2\sqrt{\alpha\tau}}$	$\text{erf}\,\frac{x}{2\sqrt{\alpha\tau}}$
0.00	0.00000	0.76	0.71754	1.52	0.96841
0.02	0.02256	0.78	0.73001	1.54	0.97059
0.04	0.04511	0.80	0.74210	1.56	0.97263
0.06	0.06762	0.82	0.75381	1.58	0.97455
0.08	0.09008	0.84	0.76514	1.60	0.97636
0.10	0.11246	0.86	0.77610	1.62	0.97804
0.12	0.13476	0.88	0.78669	1.64	0.97962
0.14	0.15695	0.90	0.79691	1.66	0.98110
0.16	0.17901	0.92	0.80677	1.68	0.98249
0.18	0.20094	0.94	0.81627	1.70	0.98379
0.20	0.22270	0.96	0.82542	1.72	0.98500
0.22	0.24430	0.98	0.83423	1.74	0.98613
0.24	0.26570	1.00	0.84270	1.76	0.98719
0.26	0.28690	1.02	0.85084	1.78	0.98817
0.28	0.30788	1.04	0.85865	1.80	0.98909
0.30	0.32863	1.06	0.86614	1.82	0.98994
0.32	0.34913	1.08	0.87333	1.84	0.99074
0.34	0.36936	1.10	0.88020	1.86	0.99147
0.36	0.38933	1.12	0.88079	1.88	0.99216
0.38	0.40901	1.14	0.89308	1.90	0.99279
0.40	0.42839	1.16	0.89910	1.92	0.99338
0.42	0.44749	1.18	0.90484	1.94	0.99392
0.44	0.46622	1.20	0.91031	1.96	0.99443
0.46	0.48466	1.22	0.91553	1.98	0.99489
0.48	0.50275	1.24	0.92050	2.00	0.995322
0.50	0.52050	1.26	0.92524	2.10	0.997020
0.52	0.53790	1.28	0.92973	2.20	0.998137
0.54	0.55494	1.30	0.93401	2.30	0.998857
0.56	0.57162	1.32	0.93806	2.40	0.999311
0.58	0.58792	1.34	0.94191	2.50	0.999593
0.60	0.60386	1.36	0.94556	2.60	0.999764
0.62	0.61941	1.38	0.94902	2.70	0.999866
0.64	0.63459	1.40	0.95228	2.80	0.999925
0.66	0.64938	1.42	0.95538	2.90	0.999959
0.68	0.66278	1.44	0.95830	3.00	0.999978
0.70	0.67780	1.46	0.96105	3.20	0.999994
0.72	0.69143	1.48	0.96365	3.40	0.999998
0.74	0.70468	1.50	0.96610	3.60	1.000000

APPENDIX B
Equation and Data Sheet

$$\frac{Q_0}{A} = \int_0^{\tau} \frac{q_0}{A} d\tau = \int_0^{\tau} \frac{k(T_0 - T_i)}{\sqrt{\pi\alpha\tau}} d\tau$$

$$\frac{T(x,\tau) - T_0}{T_i - T_0} = \text{erf}\frac{x}{2\sqrt{\alpha\tau}}$$

Reynolds numbers

Dynamic viscosity in cP [1P = 1kg/m^{2}s]; *Kinematic viscosity in cSt* [1St = 1m^2/s] $\boldsymbol{Re_d = U_m d/v}$ and/or $\boldsymbol{Re_x = U_\infty x/v}$

$p = \rho RT$; $R_{argon} = 209$ J/kg.K ; $R = \mathscr{R}/M$ (*where M = molar mass and the universal gas constant* $\mathscr{R} = 8314.5$*J/Kg.mol.K*); $C_{p,argon} = 0.3004$kJ/kg ℃; $C_{v,argon} = 0.2222$kJ/kg ℃ ;

heat capacity of aluminium $C_{aluminium} = 0.9$kJ/kg ℃ *and density of aluminium* $\rho_{al} = 2700$kg/m^3

$$\frac{\delta}{x} = \frac{4.64}{Re_x^{1/2}}$$

$$\frac{\delta}{x} = \frac{5.0}{Re_x^{1/2}}$$

$v = 17.36 \times 10^{-6}$ m^2/s

$U = m/\rho A$

$k = 0.02749$ W/m°C

$Nu_x = h_x x/k = 0.332 Re_x^{½}$ $Pr^{⅓}$

$\boldsymbol{Pr = 0.7}$

$\boldsymbol{C_{p\,air} = 1.006}$ **kJ/kg°C**

$$U = \frac{\dot{m}}{\rho A}$$

$$\frac{p_2 - p_1}{\rho} = \frac{1}{2g_c}(u_1^2 - u_2^2)$$

$$q = \bar{h}A(T_w - T_\infty)$$

θ_o/θ_i. Where $\theta_o = T_o - T_\infty$ *and* $\theta_i = T_i - T_\infty$ *etc.*

$$\frac{Q_o}{A} = \frac{\rho c V \theta_i}{A}$$

Convection Boundary Node (nodal Formulas for finite-difference calculations)

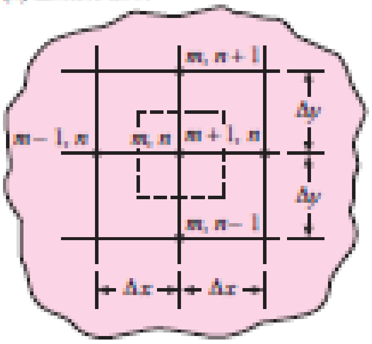
$$T_{m,n} = \frac{T_{m-1,n} + (T_{m,n+1} + T_{m,n-1})/2 + \text{Bi}\,T_\infty}{2 + \text{Bi}}$$

$$\text{Bi} = \frac{h\Delta x}{k}$$

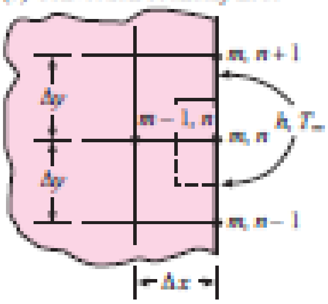
Table 3-2 | Summary of nodal formulas for finite-difference calculations. (Dashed lines indicate element volume.)†

Physical situation	Nodal equation for equal increments in x and y (second equation in situation is in form for Gauss-Seidel iteration)
(*a*) Interior node	$0 = T_{m+1,n} + T_{m,n+1} + T_{m-1,n} + T_{m,n-1} - 4T_{m,n}$ $T_{m,n} = (T_{m+1,n} + T_{m,n+1} + T_{m-1,n} + T_{m,n-1})/4$
(*b*) Convection boundary node	$0 = \frac{h\Delta x}{k} T_\infty + \frac{1}{2}(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) - \left(\frac{h\Delta x}{k} + 2\right) T_{m,n}$ $T_{m,n} = \frac{T_{m-1,n} + (T_{m,n+1} + T_{m,n-1})/2 + \mathrm{Bi}\, T_\infty}{2+\mathrm{Bi}}$ $\mathrm{Bi} = \frac{h\Delta x}{k}$
(*c*) Exterior corner with convection boundary	$0 = 2\frac{h\Delta x}{k} T_\infty + (T_{m-1,n} + T_{m,n-1}) - 2\left(\frac{h\Delta x}{k} + 1\right) T_{m,n}$ $T_{m,n} = \frac{(T_{m-1,n} + T_{m,n-1})/2 + \mathrm{Bi}\, T_\infty}{1+\mathrm{Bi}}$ $\mathrm{Bi} = \frac{h\Delta x}{k}$
(*d*) Interior corner with convection boundary	$0 = 2\frac{h\Delta x}{k} T_\infty + 2T_{m-1,n} + T_{m,n+1} + T_{m+1,n} + T_{m,n-1} - 2\left(3 + \frac{h\Delta x}{k}\right) T_{m,n}$ $T_{m,n} = \frac{\mathrm{Bi}\, T_\infty + T_{m,n+1} + T_{m-1,n} + (T_{m+1,n} + T_{m,n-1})/2}{3+\mathrm{Bi}}$ $\mathrm{Bi} = \frac{h\Delta x}{k}$

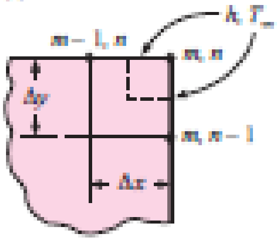
(*a*) Interior node



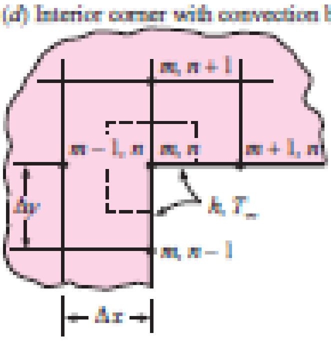
(*b*) Convection boundary node



(*c*) Exterior corner with convection boundary



(*d*) Interior corner with convection boundary



Appendix C : Heislar Charts

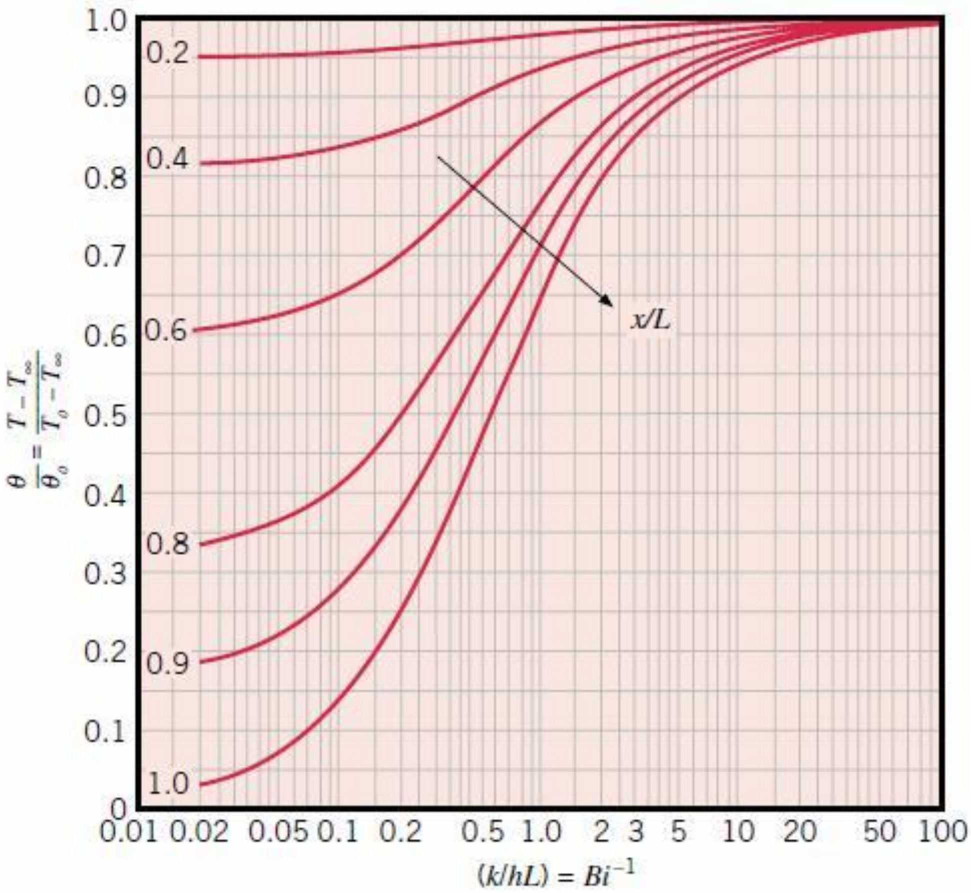


FIGURE 5S.2 Temperature distribution in a plane wall of thickness $2L$ [1]. Used with permission.

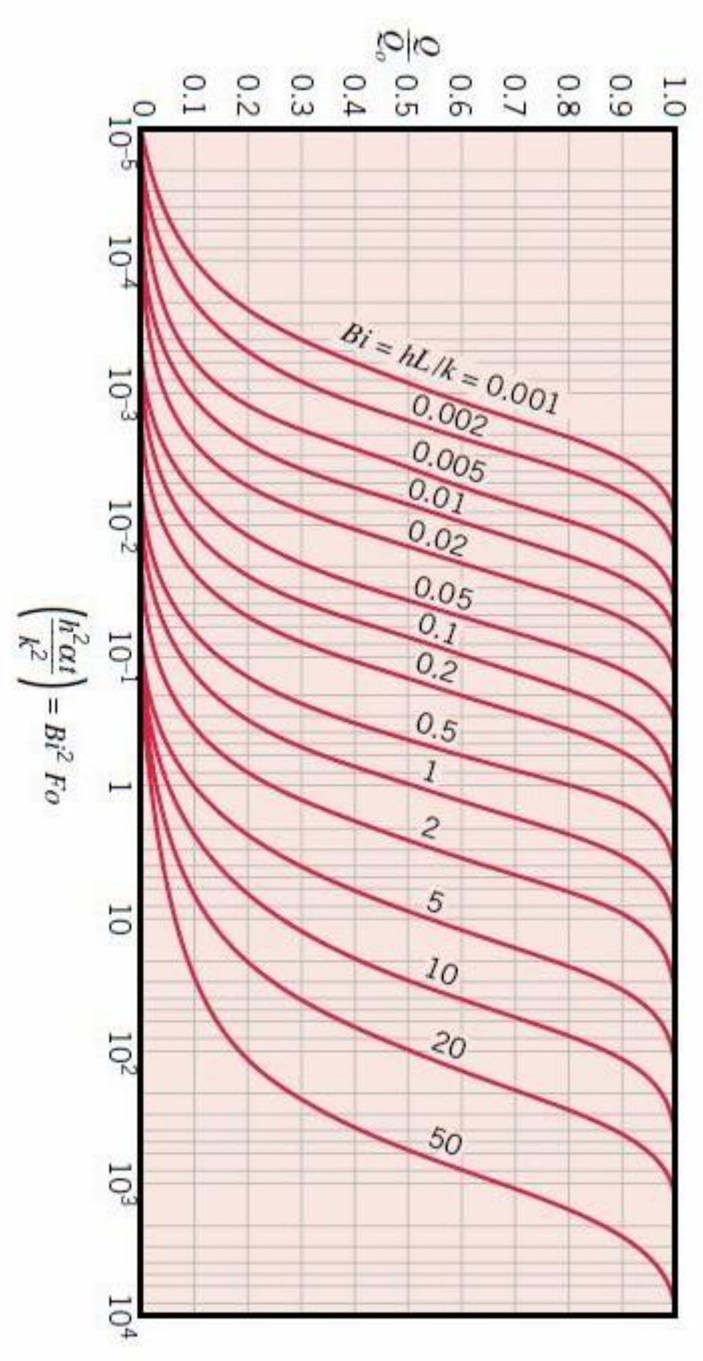


FIGURE 5S.3 Internal energy change as a function of time for a plane wall of thickness 2L [2]. Adapted with permission.

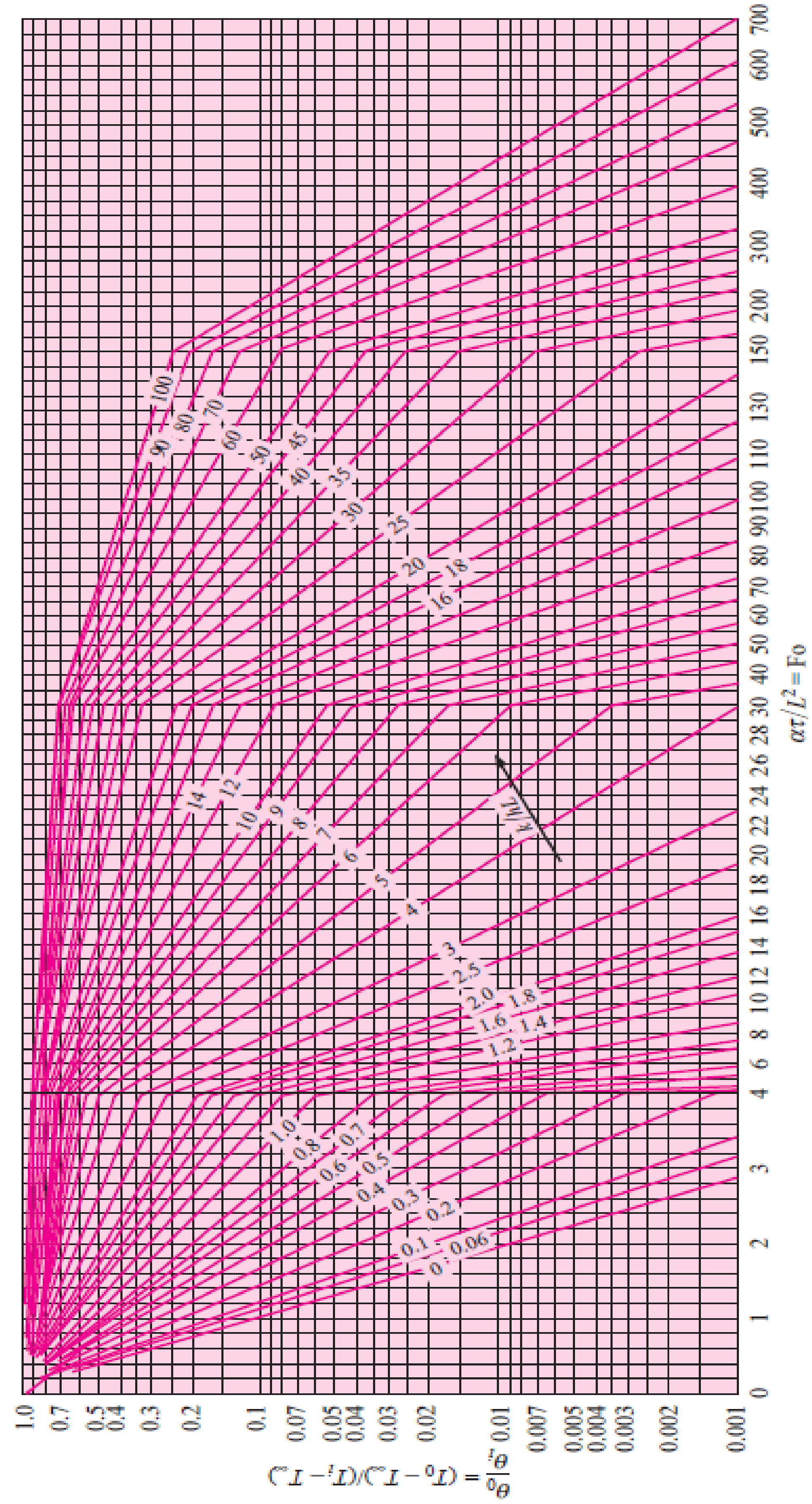


Figure 4-7 | Midplane temperature for an infinite plate of thickness $2L$: (*a*) full scale.