University of Johannesburg
College of Business and Economics
School of Economics
Main Exam
Quantitative Economics (QTE 3BB3)
Time:3 Hours
100 Marks

Assesors: Prof B.D. Simo-Kengne and Mr T Mboweni
Internal Examiner:Dr Josine Uwilingiye
External Examiner:Dr G. Aye

October 18, 2019

Instructions:

- Read the questions carefully
- This exam consists of **three** pages
- Write clearly and neatly
- Answer all questions
- Show all calculations
- Use a pen, not a pencil

Question 1

Show that $\mathbb{R}^2 = span\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}\right)$

(5 Marks)

Question 2

(a) Find the LU decomposition of matrix $A = \begin{bmatrix} 2 & 1 & -2 \\ -2 & 3 & -4 \\ 4 & -3 & 0 \end{bmatrix}$

(10 Marks)

(b) Use the solution in (a) to solve the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$

(5 Marks)

Question 3

Let

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

Find the bases for $row(A)$,$col(A)$ and $null(A)$

(10 Marks)

Question 4

Let

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Find the eigenvalues and the corresponding eigenspaces of matrix A

(10 Marks)

Question 5

Let

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 0 & 5 & -3 \\ 0 & 0 & -1 \end{bmatrix}$$

If possible find matrix P that diagonalize matrix A and find the corresponding diagonal matrix D
(10 Marks)

Question 6

Are the following statements True or False. Justify your answer

(a) If λ_1 and λ_2 are eigenvalues of a linear system of DE, a spiral sink satisfies the condition $\lambda_1 = \lambda_2 = p + iq$
(2 Marks)

(b) Consider the differential equation $dx/dt = g(x)$. the steady state x_s is unstable if and only if $g'(x_s) = 0$
(2 Marks)

(c) 2 and 0.33 are steady states for the DE $dx/dt = 3x^2(1 - x)$.
(2 Marks)

(d) If a quantity grows at a rate proportional to the distance from the threshold value, it can be modelled with logic growth model.
(2 Marks)

(e) $y = (t^2 + 5)/t$ is a solution of the DE: $y' + (y/t) = 2$
(2 Marks)

(f) A homogenous DE is also a linear DE
(2 Marks)

(12 Marks)

Question 7

Suppose that you took out college loans totalling R90,000 with interest of 8.5%. You have an online payment plan which continuously deducts money from your bank account at a rate which comes out to R16,000 per year. How long will it take you to pay off the loan? (10 Marks)

Question 8

Solve the following DE and system of linear DEs

(a) $y'' + 4y = xe^x + \cos 2x$
(6 Marks)

(b) $\dfrac{d^2y}{dt^2} - 2\dfrac{dt}{dy} + y = t + \cos t$
(6 Marks)

(c) $3xy + y^2 + (x^2 + xy)y' = 0$
(4 Marks)

(d) $9\dfrac{d^2y}{dt^2} - 12\dfrac{dt}{dy} + 4y = 0$
(4 Marks)

(e)

$$y_1' = 2y_1 + 4y_2$$
$$y_2' = y_1 + 3y_2$$

with initial conditions $y_1(0) = 0$, $y_2(0) = 0$. Note: $y_1' = \dfrac{dy_1}{dz}$, $y_2' = \dfrac{dy_2}{dz}$
(8 Marks)

(28 Marks)