University of Johannesburg
College of Business and Economics
School of Economics
Supplemetary Exam
Quantitative Economics (QTE 3BB3)
Time:3 Hours
100 Marks

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External Examiner:Dr G. Aye

Instructions:

- Read the questions carefully
- This exam consists of **three** pages
- Write clearly and neatly
- Answer all questions
- Show all calculations
- Use a pen, not a pencil

Question 1

Show that $\mathbb{R}^2 = span\left(\begin{bmatrix}3\\-2\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right)$
(5 Marks)

Question 2

(a) Find the LU decomposition of matrix $A = \begin{bmatrix}2 & 1 & 3\\4 & -1 & 3\\-2 & 5 & 5\end{bmatrix}$

(10 Marks)

(b) Use the solution in (a) to solve the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix}1\\-4\\9\end{bmatrix}$

(5 Marks)

Question 3

Let

$$A = \begin{bmatrix}1 & 1 & 0 & 1\\0 & 1 & -1 & 1\\0 & 1 & -1 & -1\end{bmatrix}$$

Find the bases for $row(A)$,$col(A)$ and $null(A)$
(10 Marks)

Question 4

Let

$$A = \begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\2 & -5 & 4\end{bmatrix}$$

Find the eigenvalues and the corresponding eigenspaces of matrix A
(10 Marks)

Question 5

Let

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{bmatrix}$$

If possible find matrix P that diagonalize matrix A and find the corresponding diagonal matrix D
(10 Marks)

Question 6

Are the following statements True or False. Justify your answer

(a) If λ_1 and λ_2 are eigenvalues of a linear system of DE, a stable node satisfies the condition $\lambda_1 < 0 < \lambda_2$.
(2 Marks)

(b) Consider the differential equation $dx/dt = g(x)$. the steady state x_s is stable if and only if $g'(x_s) > 0$
(2 Marks)

(c) 1 and 2 are unstable steady states for the DE $dy/dt = y^2 - 3y + 2$.
(2 Marks)

(d) If a population grows at a rate proportional to the population size, it can be modeled with limited growth model
(2 Marks)

(e) $x = t^2 + 5$ is a solution of the DE: $x'' + x = t^2 + 7$
(2 Marks)

(f) A separable DE is also an exact DE
(2 Marks)

(12 marks)

Question 7

A small cat-toy company is growing their business at a rate which equals about R30,000 of new profit each year. In their first year they make R50,000. They start a business bank account earning interest of 5% per year, and

deposit their profits daily. How much money should be in their account after the first 30 months?
(10 marks)

Question 8

Solve the following DE and system of linear DEs

(a) $xy^2y' = x + 1$
(5 Marks)

(b) $y'' - 2y' + y = x + \cos x$
(5 Marks)

(c) $y'' + 9y = xe^{-x}$
(5 Marks)

(d) $\dfrac{d^2y}{dt^2} - 8\dfrac{dt}{dy} + 17y = 0; \quad y(0) = 3; y(\pi) = 2$
(5 Marks)

(e)

$$y_1' = y_1 + 4y_2$$
$$y_2' = 2y_1 + 3y_2$$

with initial conditions $y_1(0) = 0$, $y_2(0) = 0$. Note: $y_1' = \dfrac{dy_1}{dt}$, $y_2' = \dfrac{dy_2}{dt}$
(8 Marks)

(28 Marks)