UNIVERSITY OF JOHANNESBURG

SCHOOL OF ECONOMICS

AUCKLAND PARK KINGSWAY CAMPUS

EXAM NOVEMBER 2019

Module Name: **Financial Economics B** **Date: November 2019**

Module Code: FNN8X02

Honours: Financial Economics **Duration: 3 H**

Mark: 90

Internal Examiner: Prof Franck Adekambi

Internal Moderator:

External Moderator: Dr Khouzeima Moutanabbir

Instructions:

1. The exam contributes 50 percent of the course mark.
2. Answer all questions.
3. This paper consists of 4 pages.
4. Please round off to 2 decimal places.

Initials & Surname: ______________________________

Student number: ______________________________

Telephone number: ______________________________

SECTION	TOTAL	MARK	EXTERNAL
Q1	12		
Q2	20		
Q3	14		
Q4	14		
Q5	20		
Q6	10		
TOTAL	90		

Q1) (12 marks)

The price of a non-dividend paying stock at time 1, S_1, is related to the price at time 0, S_0, as follows:

$S_1 = uS_0$ with probability p and $S_1 = dS_0$ with probability $(1-p)$. The continuously compounded rate of return on a risk-free asset is r.

(i) Derive an expression for the replicating portfolio for a European call option written on the stock that expires at time 1 and has a strike price of k, where $dS_0 < k < uS_0$. [5]

(ii) Show that the price of the option in (i) can be written as the discounted expected payoff under a probability measure Q. Hence find an expression for the probability, q, of an upward move in the stock price under Q. [7]

(iii) Explain the relationship between the Q probability measure in (ii) and the real world probability measure. Explain what relationship you would expect q and p to have if all investors are (a) risk averse, (b) risk seeking, or (c) risk-neutral. [5]

Q2) (20 marks)

For a two-period binomial model for stock prices, you are given:
(i) Each period is 8 months.
(ii) The current price for a non-dividend-paying stock is $80.00.
(iii) u =1.181, where u is one plus the rate of capital gain on the stock per period if the price goes up.
(iv) d = 0.890, where d is one plus the rate of capital loss on the stock per period if the price goes down.
(v) The continuously compounded risk-free interest rate is 4%.

a) **Calculate the current price of a one-year American put option on the stock with a strike price of $95.00.**

Q3) (15 marks)

A loan is being repaid with 25 annual payments of 300 each. With the 10th payment, the borrower pays an extra 1000, and then repays the balance over 10 years with a revised annual payment. The effective rate of interest is 8%.

a) **Calculate the amount of the revised annual payment.**

Q4) (14 marks)

An investor buys, for a premium of 187.06, a call option on a non-dividend paying stock whose current price is 5,000. The strike price of the call is 5,250 and the time to expiry is 6 months. The risk free rate of return over this period is 5% p.a.

The Black-Scholes formula for the value of a European call option on a non-dividend paying stock at time t can be written as:

$$c = Se^{-\delta(T-t)}\phi(d_1) - Ke^{-r(T-t)}\phi(d_2),$$

where

$$d_1 = \frac{\ln\left(S/K\right) + \left(r + \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = \frac{\ln\left(S/K\right) + \left(r - \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}},$$

K = strike price, T = time of maturity , S = price of stock at time t , r = risk-free rate, σ = volatility, δ = dividend rate.

and ϕ $(\cdot)$ = cumulative distribution function of the standard normal distribution.

(i) Calculate the price of a put option with the same time to maturity and strike price as the call. [2]

(ii) The investor buys a put option with strike price 4,750 with the same time to maturity. Calculate the price of the put option if the implied volatility were the same as that in (i).

[You need to estimate the implied volatility to within 1% p.a. of the correct value.

Hint: Use the values 0.15 and 0.18 for the linear interpolation of the volatility] [10]

(iii) Explain why the market price for the put might differ from that calculated in (ii). [2]

Q5) (25 marks)

Consider two nondividend-paying assets X and Y, whose prices are driven by the same Brownian motion Z. You are given that the assets X and Y satisfy the stochastic differential equations:

$$\frac{dX(t)}{X(t)} = 0.07dt + 0.12dZ(t)$$

$$\frac{dY(t)}{Y(t)} = Gdt + HdZ(t),\ t \geq 0.$$

where G and H are constants.
You are also given:

(i) $d\left(\ln\left[Y(t)\right]\right) = 0.06dt + \sigma dZ(t)$
(ii) The continuously compounded risk-free interest rate is 4%.
(iii) $\sigma < 0.25$

a) **Determine G.**

Q6) (10 marks)

Assume that the Black-Scholes framework holds. The price of a nondividend-paying stock is \$35.00. The price of a put option on this stock is \$7.00.

You are given:
(i) $\Delta = -0.27$
(ii) $\Gamma = 0.15$

a) **Define in words the Greeks ,** Δ **,** Γ **,** Θ **,** $Vega$ **,** $Rho = \rho$**,** $Psi = \psi$ **for an individual derivative. [6]**
b) **Explain how and why can be used in the risk management of a portfolio that is delta-hedged. [2]**
c) **Using the delta-gamma approximation, determine the price of the put option if the stock price changes to $36.50. [2]**