## SCHOOL OF ECONOMICS

**AUCKLAND PARK KINGSWAY CAMPUS**

**EXAM NOVEMBER 2019**

**Module Name**: **Financial Economics B** **Date: November 2019**

**Module Code: FNN8X02**

**Honours: Financial Economics** **Duration: 3 H**

**Mark: 90**

**Internal Examiner: Prof Franck Adekambi**

**Internal Moderator:**

**External Moderator: Dr Khouzeima Moutanabbir**

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**Instructions:**

1. The exam contributes 50 percent of the course mark.
2. Answer all questions.
3. This paper consists of 4 pages.
4. Please round off to 2 decimal places.

**Initials & Surname:** ______________________________

**Student number:** ______________________________

**Telephone number:** ____________________________

| SECTION | TOTAL | MARK | EXTERNAL |
|---|---|---|---|
| Q1 | 20 | | |
| Q2 | 15 | | |
| Q3 | 35 | | |
| Q4 | 10 | | |
| Q5 | 10 | | |
| TOTAL | 90 | | |

**Q1) (20 marks)**

A non-dividend paying stock has a current price of R100. In any unit of time the price of the stock is expected to increase by 10% or decrease by 5%. The continuously compounded risk-free interest rate is 4% per unit of time.

A European call option is written with a strike price of R103 and is exercisable after two units of time, at t = 2.

Establish, using a binomial tree, the replicating portfolio for the option at the start and end of the first unit of time, i.e. at t = 0, 1. Hence, calculate the value of the option at t = 0. [20]

**Q2) (15 marks)**

A loan is being repaid with 25 annual payments of 300 each. With the 10th payment, the borrower pays an extra 1000, and then repays the balance over 10 years with a revised annual payment. The effective rate of interest is 8%.

a) **Calculate the amount of the revised annual payment.**

**Q3) (35 marks)**

Suppose that a stock price is modelled by a two period recombining binomial model, where each period is six months, with the following parameters:

risk free interest rate r = 3% p.a.; volatility = 10% p.a.; initial share price $S_0 = 100$.

The up step is given by $u = e^{\left(\frac{0.1}{\sqrt{2}}\right)}$.

Consider a European call option, expiring in one year's time with exercise price 90.

(i) Construct a two period recombining tree, showing the share price at each node, and derive the risk neutral probabilities assuming that no dividends are payable. [5]

(ii) Using the binomial tree in (i), determine the value of the option. [5]

Assume now that a dividend of 20 is payable immediately before the expiry of the option.

(iii) (a) Write down the share prices at the final three nodes of the binomial tree in (ii) assuming the same total return as before. [5]

(b) Write down the payoff at each of these three nodes of the option . [5]

(iv) Using the risk neutral probabilities from (i) determine the value of the payoff under the option at the end of the first time period, assuming the share price has jumped up in the first time period . [2]

(v) Suppose now that the option is American, not European.

(a) Determine the payoff at the node described in (iv) (i.e. at the end of the first time period, assuming the share price has moved up in the first time period). [5]

(b) Using (a), determine whether the holder of the American option should exercise early at the node described in (iv). [5]

(c) State with reasons whether the value at time zero of the American option is either:

(1) strictly less than

(2) greater than or equal to

(3) strictly greater than

the value of the European option. [3]

**Q4) (10 marks)**

(i) Explain the terms delta, gamma, Vega, Psi, Rho and theta of an option. [6]

(ii) Describe, including a numerical example, the concept of delta hedging of options. [4]

**Q5) (10 marks)**

The Black-Scholes formula for the value of a European call option on a non-dividend paying stock at time $t$ can be written as:

$$c = Se^{-\delta(T-t)}\phi(d_1) - Ke^{-r(T-t)}\phi(d_2),$$

where

$$d_1 = \frac{\ln\left(S/K\right) + \left(r + \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = \frac{\ln\left(S/K\right) + \left(r - \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}},$$

$K$ = strike price, $T$ = time of maturity , $S$ = price of stock at time $t$, $r$ = risk-free rate, $\sigma$ = volatility, $\delta$ = dividend rate.

and $\phi$ $(\cdot)$ = cumulative distribution function of the standard normal distribution.

(a) Using the Black-Scholes formula show that the call price, $c$, is the maximum of $S - Ke^{-r(T-t)}$ or zero, depending on the strike price, when σ tends to zero. [10]