University of Johannesburg
School of Economics
Main Exam
Econometrics 2B (ECM02B2-EKM2B01)
Time:3 Hours
80 Marks

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November 2019

- Read the questions carefully
- This exam consists of **three** pages
- Write clearly and neatly
- Answer all questions
- Show all calculations
- Use a pen, not a pencil

Question 1

Let Y be a Poisson random variable with probability density function

$$p(y) = \frac{\lambda^y}{y!}e^{-\lambda}, \quad y = 0, 1, 2, \cdots, \lambda > 0$$

(a) Show that the moment genarting function for this distribution is given by $m(t) = e^{\lambda(e^t-1)}$
(6 Marks)

(b) Use the solution in (a) to find the mean and the variance for random variable Y
(4 Marks)

(10 Marks)

Question 2

The probability distribution of the random variable Y is given by

$$f(y) = \begin{cases} c(2-y), & 0 \leq y \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find

(a) c
(4 Marks)

(b) Find $F(y)$
(6 Marks)

(c) Graph $f(y)$ and $F(y)$
(4 Marks)

(d) Find $P(1 \leq Y \leq 2)$
(4 Marks)

(e) Find the mean and the variance of Y
(6 Marks)

(24 Marks)

Question 3

A random variable Y has a *uniform distribution* over the interval (θ_1, θ_2) given by

$$f(y) = \begin{cases} \frac{1}{\theta_2 - \theta_1}, & \theta_1 \leq y \leq \theta_2 \\ 0, & \text{elsewhere} \end{cases}$$

Show that the mean and variance are given by

$$\mu = E(Y) = \frac{\theta_1 + \theta_2}{2} \quad \text{and} \quad \sigma^2 = V(Y) = \frac{(\theta_2 - \theta_1)^2}{12}$$

(10 marks)

Question 4

Let Y_1 and Y_2 be random variables with joint probability distribution

$$f(y_1, y_2) = \begin{cases} 4y_1y_2, & 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Are Y_1 and Y_2 independent
(6 Marks)

Question 5

Let $Y_1, Y_2, \cdots, Y_n$ denote a random sample from the normal distribution with mean $\mu = 0$ and variance σ^2 with density function

$$f(y) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2\right], & -\infty < y < \infty \\ 0, \quad elsewhere \end{cases}$$

Find the method-of moments estimator of σ^2
(6 Marks)

Question 6

Let $Y_1, Y_2, \cdots, Y_n$ denote a random sample of size n from the Poisson distribution with mean λ. Consider the two estimators of λ given by

$$\hat{\lambda}_1 = \frac{Y_1 + Y_2}{2} \quad \text{and} \quad \hat{\lambda}_2 = \bar{Y}$$

Find the efficiency of $\hat{\lambda}_1$ relative to $\hat{\lambda}_2$
(4 Marks)

Question 7

Show that the mean squared error of an estimator $\hat{\theta}$ for parameter θ is given by $MSE(\hat{\theta}) = V(\hat{\theta}) + B(\hat{\theta})^2$ where $V(\hat{\theta})$ is the variance and $B(\hat{\theta})$ is the bias of the estimator $\hat{\theta}$
(6 Marks)

Question 8

Let $Y_1, Y_2, \cdots, Y_n$ denote a random sample a Poisson denstity function

(a) Find the maximum likelihood estimator (MLE) $\hat{\lambda}$ for λ
(6 Marks)

(b) Find the expected value and variance of $\hat{\lambda}$
(4 Marks)

(c) Show that the estimator $\hat{\lambda}$ is consistent for λ
(4 Marks)

(14 Marks)