PROGRAM

## SUBJECT

CODE

DATE
: SUMMER EXAMINATION 2019 21 NOVEMBER 2019

WEIGHT : $40: 60$

TOTAL MARKS : $100=100 \%$

EXAMINER : MR S.P SIMELANE
MODERATOR : MR. J. NWAMBA
NUMBER OF PAGES
5 EXAMINATION PAGES (Including cover page and Formula Sheet)

## INSTRUCTIONS

1. PLEASE ANSWER ALL QUESTIONS NEATLY.
2. SHOW ALL CALCULATIONS
3. ANSWERS WITHOUT UNITS WILL BE PENALIZED
4. NUMBER YOUR ANSWERS STRICTLY ACCORDING TO THE QUESTIONS
5. $\quad \gamma_{\text {(air) }}=1.4 ; \mathrm{R}_{\text {(air) }}=0.287 \mathrm{~kJ} / \mathrm{kg} . \mathrm{K} ; \mathrm{C}_{\mathrm{p} \text { (air) }}=1,005 \mathrm{~kJ} / \mathrm{kgK} ; \mathrm{g}=9.81 \mathrm{~m} / \mathrm{s}^{2} ; \gamma_{\mathrm{g}}=1.333$;
$C_{\mathrm{pg}}=1.147 \mathrm{~kJ} / \mathrm{kgK}$.
6. ALWAYS DRAW DIADRAMS

REQUIREMENTS: CALCULATORS ARE PERMITTED

## QUESTION 1

1.1 You have just been hired as a pumps specialist and your first job is to pump water from one large reservoir to another large reservoir that is at a higher elevation. The free surfaces of both reservoirs are exposed to atmospheric pressure, as sketched in the Figure below where the dimensions and minor loss coefficients are also provided.

$$
\begin{aligned}
z_{2}-z_{1} & =7.85 \mathrm{~m} \text { (elevation difference) } \\
D & =2.03 \mathrm{~cm} \text { (pipe diameter) } \\
K_{L, \text { entrance }} & =0.50 \text { (pipe entrance) } \\
K_{L, \text { valve }} & =17.5 \text { (valve) } \\
K_{L, \text { elbow }} & =0.92 \text { (each elbow-there are } 5) \\
K_{L, \text { exit }} & =1.05 \text { (pipe exit) } \\
L & =176.5 \mathrm{~m} \text { (total pipe length) }
\end{aligned}
$$



The expression in the pump's performance curve contains the shutoff head $H_{0}=24.4 \mathrm{~m}$ of water column and the coefficient $a=0.0678 \mathrm{~m} / \mathrm{Lpm}^{2}$. Estimate the capacity delivered by the pump if the coefficient of pipe friction is 0.1024 .
1.2 You are now asked to explore another option where the free surface of the inlet reservoir in problem 1.1 is increased by 3 m , making the elevation difference smaller. All the constants and parameters are left unchanged except for the elevation difference. Compare the volume flow rate for this case and compare with the result of the previous setup. Discuss.
1.3 Your supervisor then asks you to find a replacement pump that will increase the flow rate through the piping system of Problem 1.1 above by a factor of 2 or greater. You look through some online brochures, and find a pump with the performance data shown in the Table below.

| $\dot{V}, \mathrm{Lpm}$ | $H, \mathrm{~m}$ |
| :--- | :--- |
| 0 | 46.5 |
| 5 | 46 |
| 10 | 42 |
| 15 | 37 |
| 20 | 29 |
| 25 | 16.5 |
| 30 | 0 |

All dimensions and parameters remain the same as in 1.1, only the pump is changed.
1.3.1 Perform a least-squares curve fit (regression analysis) of $H_{\text {available }}$ versus $\dot{V}^{2}$, where $\dot{V}$ is the flow rate, and establish the best-fit values of coefficients $H_{0}$ and $a$ that translate the tabulated data of the Table above into the appropriate parabolic expression.
1.3.2 Use the expression obtained in part 1.3.1 to estimate the operating volume flow rate of the new pump if it were to replace the existing pump, all else being equal. Compare to the result of problem 1.1 and discuss. Have you achieved your goal?

## QUESTION 2

The information below is that of a preliminary analysis of a Francis radial-flow hydroturbine that runs at 120 rpm , with the volume flow rate at design conditions of $80.0 \mathrm{~m}^{3} / \mathrm{s}$, where location 2 is the inlet and location 1 is the outlet. You are required to assess whether the turbine has forward or reverse swirl.
$r_{2}=2.00 \mathrm{~m}, r_{1}=1.30 \mathrm{~m}, b_{2}=0.85 \mathrm{~m}$, and $\mathrm{b}_{1}=2.10 \mathrm{~m}$, runner blade angles $\beta_{2}=71.4^{\circ}$ and $\beta_{1}$ $=15.3^{\circ}$ at the turbine inlet and outlet, respectively. $\rho_{\text {water }}=998 \mathrm{~kg} / \mathrm{m}^{3}$


Also predict the power output (MW) and the required net head (m) while neglecting irreversible losses.
[15]

## QUESTION 3

A turbomachinary company brings you in to use your understanding of free-vortex conditions to help work out the flow angles and the degree of reaction at the hub of one particular axial flow compressor. This compressor has a tip diameter of 0.95 m and a hub diameter of 0.85 m and rotates at 5000 rpm . If air enters at an angle of $28^{\circ}$ and with relative velocity angle of $56^{\circ}$, and for ease of analysis at this initial design stage you suggest $50 \%$ degree of reaction at the mean radius, you also need to work out how much air and how much power the compressor will need to do its job.
[18]

## QUESTION 4

Another turbomachine needs your attention, and this time it is a single-stage axial gas turbine with a total-to-static efficiency of 0.85 , and mean blade speed is $480 \mathrm{~m} / \mathrm{s}$. If the nozzle efflux angle is $68^{\circ}$, the stagnation temperature and stagnation pressure at stage inlet are $800^{\circ} \mathrm{C}$ and 4 bar, respectively, and the gas is exhausted at a static pressure of 1 bar, and based on how much specific work the turbine can produce, estimate the axial velocity which is constant through the stage, as well as the total-to-total efficiency. What is the degree of reaction of this turbine?

## QUESTION 5

Basing your analysis on static conditions and an appropriate slip factor, establish the adiabatic efficiency of a 5000 rpm centrifugal compressor with an impeller diameter of 1 m and a static pressure ratio of 2.2 , dealing with $28 \mathrm{~kg} / \mathrm{s}$ of air. Also indicate the impeller exit temperature and how much power is needed.
[9]

## QUESTION 6

An inward flow radial gas turbine rotating at $38140 \mathrm{rev} / \mathrm{min}$ is to be "cold" tested in a laboratory where an air supply is available only at the stagnation conditions of 200 kPa and 102 ${ }^{\circ} \mathrm{C}$. The aim is to work out, for the "cold" test, the equivalent mass flow rate and the speed of rotation. A rotor diameter of 23.76 cm is proposed to operate with a gas mass flow of $1.0 \mathrm{~kg} / \mathrm{s}$, and dynamically similar conditions between those of the laboratory and the projected design are assumed. At the design condition the inlet stagnation pressure and temperature are set at 300 kPa and $727^{\circ} \mathrm{C}$, respectively.
At $\mathrm{T}_{01}=1000 \mathrm{~K} \mu=4.153 \times 10^{-5} \mathrm{~kg} / \mathrm{m} . \mathrm{s} ; \mathrm{T}_{01}=375, \mu=2.181 \times 10^{-5} \mathrm{~kg} / \mathrm{m} . \mathrm{s}$
6.1 For ease of analysis, assume the gas properties are the same as for air and work out the equivalent mass flow rate and the speed of rotation.
6.2 If the Reynolds number is defined as $R e=\rho_{01} N D^{2} / \mu_{01}$, compare the Reynolds numbers for the two conditions.
6.3 Dealing with the prescribed "hot" design point conditions, in which the gas leaves the exducer directly to the atmosphere at a pressure of 100 kPa and without swirl, and the absolute flow angle at rotor inlet is $72{ }^{\circ} \mathrm{C}$ to the radial direction, the relative velocity at the mean radius of the exducer (which is one half of the rotor inlet radius) was found to be twice the rotor inlet relative velocity. Use a nozzle enthalpy loss coefficient of 0.06 to determine the total-to-static efficiency of the turbine, the static temperature and pressure at the rotor inlet as well as the axial width of the passage at inlet to the rotor.
$\dot{m}=\rho A C_{x} \quad P=g H Q=\rho g H \dot{V} \quad h_{L}=f \frac{L}{D} \frac{V^{2}}{2 g}$
$P=g H Q=\rho g H \dot{V} \quad \Delta W=C_{p} \Delta T_{0}=U \Delta c_{y}=U \Delta V_{y} \quad U=\frac{\pi D N}{60}=\omega r$
$\frac{P_{1}}{\rho_{1} g}+\frac{V_{1}^{2}}{2 g}+z_{1}+h_{\text {pump }, u}=\frac{P_{2}}{\rho_{2} g}+\frac{V_{2}^{2}}{2 g}+z_{2}+h_{\text {turbine } e}+h_{L} \quad\left(\frac{\dot{m}}{D^{2} P_{01}}\right) \sqrt{R T_{01}}$
$\dot{m}\left(\frac{P_{1}}{\rho}+\alpha_{1} \frac{V_{1}^{2}}{2}+g z_{1}\right)+\dot{W}_{\text {pump }}=\dot{m}\left(\frac{P_{2}}{\rho}+\alpha_{2} \frac{V_{2}^{2}}{2}+g z_{2}\right)+\dot{W}_{\text {turbine }}+\dot{E}_{\text {mech,loss }}$
$T_{0}=T+\frac{C^{2}}{2 C_{p}}$
$\rho=\frac{p}{R T} \quad \psi=\Delta h_{0} / U^{2}$
$\psi=\lambda \varphi\left(\tan \beta_{1}-\tan \beta_{2}\right)$
$\Delta W=h_{03}-h_{01}=\sigma_{s} U_{2}^{2} \quad M=\frac{C}{\sqrt{\gamma R T}}$
$R_{s}=\left[1+\frac{\eta_{s} \Delta T_{0 s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$
$\frac{p_{2}}{p_{02}}=\left[\frac{T_{2}}{T_{02}}\right]^{\frac{\gamma}{\gamma-1}}$
$\frac{C_{p}}{R}=\frac{\gamma}{\gamma-1}$
$C_{w} r=$ const
$\Lambda=\frac{c_{a}}{2 U}\left(\tan \beta_{1}+\tan \beta_{2}\right)$
$T_{3}-T_{3 s}=\frac{\lambda_{R} w_{3}^{2}}{2 c_{p}}$
$\psi=\phi\left(\tan \beta_{1}-\tan \beta_{2}\right) \quad \psi=\phi\left(\tan \beta_{2}+\tan \beta_{3}\right) \quad \Lambda=\frac{\phi}{2}\left(\tan \beta_{3}-\tan \beta_{2}\right)$
$h_{01}=h_{1}+\frac{C_{1}^{2}}{2}$
$\lambda_{N}=\frac{C_{p}\left(T_{2}-T_{2 s}\right)}{\frac{1}{2} C_{2}^{2}} \quad A=2 \pi r_{m} h$
$\frac{T_{02}}{T_{01}}=\left[\frac{P_{02}}{P_{01}}\right]^{\frac{(\gamma-1)}{\gamma \eta_{p}}} \quad T_{02}-T_{2}=\frac{C_{2}^{2}}{2 C_{p}} \quad \Lambda=\frac{T_{2}-T_{3}}{T_{1}-T_{3}}$
$\Delta W=C_{p}\left(T_{02}-T_{01}\right)=\lambda U C_{a}\left(\tan \beta_{1}-\tan \beta_{2}\right)$
$\Delta W=C_{p g}\left(\Delta T_{0 s}\right)=U\left(C_{w 3}+C_{w 2}\right)$
$\frac{U}{c_{a}}=\tan \beta_{3}-\tan \alpha_{3}=\tan \alpha_{2}-\tan \beta_{2} \quad \Delta h_{0 s}=\frac{1}{2} C_{0}^{2}$
$\Lambda=\left(h_{2}-h_{3}\right) /\left(h_{1}-h_{3}\right)$

