UNIVERSITY
JOHANNESBURG

| PROGRAM | :BACCALAUREUS INGENERIAE |
| :---: | :---: |
|  | CIVIL ENGINEERING SCIENCE |
| SUBJECT | STRENGTH OF MATERIALS 2B |
| CODE | SMC2B21 |
| DATE | SSA EXAMINATION DECEMBER/JANUARY 2019 |
| DURATION | 3 Hours |
| WEIGHT | $50: 50$ |
| TOTAL MARKS | 100 |
| ASSESSOR | MR P VAN TONDER |
| MODERATOR | PROF EKOLU |
| NUMBER OF PAGES | 4 PAGES |
| INSTRUCTIONS | ONLY ONE POCKET CALCULATOR PER CANDIDATE MAY BE USED |
| REQUIREMENTS | : NONE |

## INSTRUCTIONS TO STUDENTS

HAND IN THE QUESTION PAPER AND ANSWER BOOKS.

## QUESTION 1

Prove that for any stress block as shown in figure 1, rotated anti-clockwise through an angle $\theta$, the maximum stresses and angles for these maximum stresses are:
$\sigma_{1,2}=\frac{\sigma_{x}+\sigma_{y}}{2} \pm \sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau_{x y}^{2}}$

$$
\tan 2 \theta_{p}=\frac{\tau_{x y}}{\left(\sigma_{x}-\sigma_{y}\right) / 2}
$$

$\tau_{\text {max in-plane }}=\sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau_{x y}^{2}}$

$$
\tan 2 \theta_{s}=-\frac{\left(\sigma_{x}-\sigma_{y}\right) / 2}{\tau_{x y}}
$$

Further prove that the radius and center point for a Mohr circle of stress for figure 1 are:
$\sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau_{x y}^{2}}$

$$
\left(\frac{\sigma_{x}+\sigma_{y}}{2} ; 0\right)
$$



Figure 1

## QUESTION 2

The specimen in figure 2 represents a filament reinforced matrix system made from plactic(matrix) and glass(fiber). If there are $n$ fibers, each having a cross-sectional area of $A_{f}$ and modulus of elasticity of $E_{f}$, embedded in a matrix having a cross-sectional area of $A_{m}$ and modulus of elasticity of $E_{m}$, determine the stress in the matrix and the fiber when the force P is applied to the specimen.


Figure 2

A point on a thin plate is subjected to the strains as shown in figure 3. Determine (a) the strains if the x -axis is rotated $25^{\circ}$ clockwise (use the method of Mohr) and (b) the strains if the y -axis is rotated $50^{\circ}$ counter-clockwise (use the stress transformation equations). ( $\varepsilon_{\mathrm{x}}=-$ $780 \times 10^{-6}, \varepsilon_{y}=400 \times 10^{-6}$ and $\gamma_{\mathrm{xy}}=500 \times 10^{-6}$ ) Which block elongates the most? Which block is squashed the most?


Figure 3

## QUESTION 4

Figure 4 shows a stress block experiences by a structural element. . Determine the principle stresses and in-plane maximum shear stresses. Hence determine the orientation and stresses of the stress block where the shear stress is half of the in-plane maximum shear stress. Use the method of Mohr Circles.


Figure 4

## QUESTION 5

The bolt in figure 5 is made of aluminum alloy $(\mathrm{E}=73.1 \mathrm{GPa})$ and is tightened so it compresses a cylindrical tube made of magnesium alloy $(\mathrm{E}=44.7 \mathrm{GPa})$. The tube has an outer radius of 10 mm and it is assumed that both the inner radius of the tube and the radius of the bolt are 5 mm . The washers at the top and bottom of the tube are considered to be rigid and have a negligible thickness. Initially the nut is hand tightened snuggle; then, using a wrench, the nut is further tightened one-half turn. If the bolt has 1 thread per mm, determine the stress in the bolt.


Figure 5

Information Sheet
$\delta=\frac{P L}{A E}$
$\delta_{T}=\alpha \Delta T L$
$\tau=\frac{T \rho}{J}$
$\phi=\frac{T L}{J G}$
$J=\frac{\pi}{2} r^{4}$
$J=\frac{\pi}{2}\left(r_{o}^{4}-r_{i}^{4}\right)$
$\tau=\frac{V Q}{I t}$
$\sigma=\frac{P}{A}$
$\sigma=\frac{M y}{I}$
$\sigma_{1}=\frac{p r}{t}$
$\sigma_{2}=\frac{p r}{2 t}$
$\sigma_{x^{\prime}}=\frac{\sigma_{x}+\sigma_{y}}{2}+\frac{\sigma_{x}-\sigma_{y}}{2} \cos 2 \theta+\tau_{x y} \sin 2 \theta$
$\sigma_{y^{\prime}}=\frac{\sigma_{x}+\sigma_{y}}{2}-\frac{\sigma_{x}-\sigma_{y}}{2} \cos 2 \theta-\tau_{x y} \sin 2 \theta$
$\tau_{x^{\prime} y^{\prime}}=-\frac{\sigma_{x}-\sigma_{y}}{2} \sin 2 \theta+\tau_{x y} \cos 2 \theta$
$\sigma_{1,2}=\frac{\sigma_{x}+\sigma_{y}}{2} \pm \sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau_{x y}^{2}}$
$\tan 2 \theta_{p}=\frac{\tau_{x y}}{\left(\sigma_{x}-\sigma_{y}\right) / 2}$
$\tau_{\max \text { in -plane }}=\sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau_{x y}^{2}}$
$\tan 2 \theta_{s}=-\frac{\left(\sigma_{x}-\sigma_{y}\right) / 2}{\tau_{x y}}$
$\tau_{\text {average }}=\frac{\sigma_{x}+\sigma_{y}}{2}$
$\tau_{a b s \max }=\frac{\sigma_{1}}{2}$
$\tau_{a b s \max }=\frac{\sigma_{1}-\sigma_{2}}{2}$
$\varepsilon_{x^{\prime}}=\varepsilon_{x} \cos ^{2} \theta+\varepsilon_{y} \sin ^{2} \theta+\gamma_{x y} \sin \theta \cos \theta$
$\varepsilon_{x^{\prime}}=\frac{\varepsilon_{x}+\varepsilon_{y}}{2}+\frac{\varepsilon_{x}-\varepsilon_{y}}{2} \cos 2 \theta+\frac{\gamma_{x y}}{2} \sin 2 \theta$
$\frac{\tau_{x^{\prime} y^{\prime}}}{2}=-\frac{\varepsilon_{x}-\varepsilon_{y}}{2} \sin 2 \theta+\frac{\gamma_{x y}}{2} \cos 2 \theta$
$\varepsilon_{y^{\prime}}=\frac{\varepsilon_{x}+\varepsilon_{y}}{2}-\frac{\varepsilon_{x}-\varepsilon_{y}}{2} \cos 2 \theta-\frac{\gamma_{x y}}{2} \sin 2 \theta$
$\tan 2 \theta_{p}=\frac{\gamma_{x y}}{\left(\varepsilon_{x}-\varepsilon_{y}\right)}$
$\varepsilon_{1,2}=\frac{\varepsilon_{x}+\varepsilon_{y}}{2} \pm \sqrt{\left(\frac{\varepsilon_{x}-\varepsilon_{y}}{2}\right)^{2}+\left(\frac{\gamma_{x y}}{2}\right)^{2}}$
$\tan 2 \theta_{s}=-\frac{\left(\varepsilon_{x}-\varepsilon_{y}\right)}{\gamma_{x y}}$
$\frac{\gamma_{\text {max } \text { in-plane }}}{2}=\sqrt{\left(\frac{\varepsilon_{x}-\varepsilon_{y}}{2}\right)^{2}+\left(\frac{\gamma_{x y}}{2}\right)^{2}}$
$\varepsilon_{\text {avg }}=\frac{\varepsilon_{x}+\varepsilon_{y}}{2}$
$\gamma_{a b s \max }=\varepsilon_{1}$
$\varepsilon_{a b s \max }=\varepsilon_{1}-\varepsilon_{2}$

## Shapes

## Quarter-circular area

## Semicircular area


$\frac{4 r}{3 \pi} \quad \frac{4 r}{3 \pi} \quad \frac{\pi r^{2}}{3 \pi}$
$0 \quad \frac{4 r}{3 \pi} \quad \frac{\pi r^{2}}{2}$

