$\frac{\text { UNIVERSITY }}{\text { JOHANNESBURG }}$

| PROGRAM | BACCALAUREUS INGENERIAE MECHANICAL ENGINEERING |
| :---: | :---: |
| SUBJECT | STRENGTH OF MATERIALS 3B |
| CODE | SLR 3B21 / SLRBCB3 |
| DATE | SUPPLEMENTARY EXAMINATION JANUARY 2020 |
| DURATION | 3 HOURS |
| WEIGHT | 50:50 |
| TOTAL MARKS | 100 |
| EXAMINER | DR D. M. MADYIRA |
| MODERATOR | PROF R. F. LAUBSCHER |
| NUMBER OF PAGES | 6 PAGES |
| INSTRUCTIONS | QUESTION PAPERS MUST NOT BE HANDED IN. |
| REQUIREMENTS | ANSWER SHEETS |

## INSTRUCTIONS TO CANDIDATES:

1. Answer all questions.
2. Explain answers and give all the necessary steps to arrive at the answer - simply giving the answer is not sufficient.
3. The examination is not an open book exam. All required formulae are given in the formulae sheet.
4. Do all the questions in the answer scripts.

## QUESTION 1 [25]

A thin-walled tubular section is semi-circular in shape as shown in Figure Q1. Find the maximum torque that the section can carry if the maximum shear stress is limited to 90 MPa. For this torque, determine the angle of twist per unit length. G $=9 \mathrm{GPa}$. Consider the thickness to be constant throughout.


Figure Q1: Curved support bracket

## QUESTION 2 [25]

A compound cylinder is formed by shrinking one tube on to another, the inner and outer diameters of the outer tube being 120 mm and 180 mm respectively and that of the inner tube being 60 mm and 120 mm respectively. After shrinking, the radial pressure at the common surface is 30 MPa . If the cylinder is subjected to an internal pressure of 80 MPa , determine the final stresses set up at the various surfaces of the cylinder. What is the resultant pressure at the common surface?

## QUESTION 3 [25]

A brass sleeve $S$ is fitted over a steel bolt $B$ (see Figure Q3), and the nut is tightened until it is just snug. The bolt has a diameter $\mathrm{d}_{\mathrm{B}}=25 \mathrm{~mm}$, and the sleeve has inside and outside diameters $\mathrm{d}_{1}=26 \mathrm{~mm}$ and $\mathrm{d}_{2}=36 \mathrm{~mm}$, respectively. Calculate the temperature rise $\Delta \mathrm{T}$ that is required to produce a compressive stress of 25 MPa in the sleeve. (Use material properties as follows: for the sleeve, $\alpha_{\mathrm{S}}=21 \times 10^{-6} /{ }^{\circ} \mathrm{C}$ and $\mathrm{E}_{\mathrm{S}}=100 \mathrm{GPa}$; for the bolt, $\alpha_{\mathrm{B}}=10 \times 10^{-6} /{ }^{\circ} \mathrm{C}$ and $E_{B}=200 \mathrm{GPa}$.)


Figure Q1: Bolt and sleeve assembly

## QUESTION 4 [25]

An I-section beam is built up of a $200 \times 10 \mathrm{~mm}$ web plate with $120 \times 20 \mathrm{~mm}$ flange plates secured by rivets through $40 \times 40 \times 6 \mathrm{~mm}$ angle sections as shown in Figure Q4. Determine the maximum uniformly distributed load which can be applied over a span of 10 m if the permissible bending stress is 90 MPa (assume simply supported). Also find the pitch of the rivets. The rivets are 10 mm diameter with a permissible shear stress of 70 MPa .


Figure Q4: Fabricated beam section

## Formula Sheet

## Buckling Equations

$$
\begin{array}{cc}
P_{c r}=\frac{\pi^{2} E I}{4 L^{2}} \quad P_{c r}=\frac{\pi^{2} E I}{L^{2}} \quad P_{c r} \approx \frac{2 \pi^{2} E I}{L^{2}} \quad P_{c r}=\frac{4 \pi^{2} E I}{L^{2}} \\
\sigma_{\max }=\frac{P}{A}\left[1+\frac{e c}{r^{2}} \sec \left(\frac{L}{2 r} \sqrt{\frac{P}{E A}}\right)\right]
\end{array}
$$

## Initially Curved Sections

$$
R=A / \int_{A} \frac{d A}{r}
$$

$$
\sigma=\frac{M(R-r)}{r A(\bar{r}-R)}
$$

## Shear Stresses in Bending

$$
\tau_{x y}=\tau_{y x}=\frac{Q}{b \cdot I} \cdot \int_{A} y \cdot d A=\frac{Q \cdot A \cdot \bar{y}_{A}}{b \cdot I}
$$

## Springs

$$
\begin{aligned}
\tau_{\max } & =\frac{8 \cdot F \cdot D}{\pi \cdot d^{3}}+\frac{4 F}{\pi \cdot d^{2}} & y=\alpha \cdot \frac{D}{2}=\frac{8 \cdot F \cdot D^{3} \cdot N}{d^{4} \cdot G} & k=\frac{d^{4} \cdot G}{8 \cdot D^{3} \cdot N} \\
\sigma & =\frac{M}{I / c}+\frac{F}{A}=K \cdot \frac{32 \cdot F \cdot r_{m}}{\pi \cdot d^{3}}+\frac{4 \cdot F}{\pi \cdot d^{2}} & K \approx \frac{r_{m}}{r_{i}} & N=N_{T}-N_{D}
\end{aligned}
$$

## Thick Cylinders

$$
\begin{array}{cl}
\sigma_{r}=\frac{1}{k^{2}-1} \cdot\left[p_{i} \cdot\left(1-\frac{r_{o}^{2}}{r^{2}}\right)-p_{o} \cdot k^{2} \cdot\left(1-\frac{r_{i}^{2}}{r^{2}}\right)\right] & \sigma_{r}=\frac{p_{i}}{k^{2}-1} \cdot\left(1-\frac{r_{o}^{2}}{r^{2}}\right) \\
\sigma_{\theta}=\frac{1}{k^{2}-1} \cdot\left[p_{i} \cdot\left(1+\frac{r_{o}^{2}}{r^{2}}\right)-p_{o} \cdot k^{2} \cdot\left(1+\frac{r_{i}^{2}}{r^{2}}\right)\right] & \sigma_{\theta}=\frac{p_{i}}{k^{2}-1} \cdot\left(1+\frac{r_{o}^{2}}{r^{2}}\right) \\
k=\frac{r_{o}}{r_{i}} & \sigma_{r}=A-\frac{B}{r^{2}}
\end{array}
$$

$$
\begin{gathered}
\delta=-u^{\prime}+u^{\prime \prime}=r_{m} \cdot\left(\varepsilon_{\theta}^{\prime \prime}-\varepsilon_{\theta}^{\prime}\right) \\
\varepsilon_{\theta}^{\prime \prime}=\frac{1}{E}\left(\sigma_{\theta}^{\prime \prime}-v \sigma_{r}^{\prime \prime}\right) \quad \text { outer cylinder } \\
\varepsilon_{\theta}^{\prime}=\frac{1}{E}\left(\sigma_{\theta}^{\prime}-v \sigma_{r}^{\prime}\right) \quad \text { inner cylinder }
\end{gathered}
$$

## Rotating Components

$$
\begin{array}{cc}
\frac{d \sigma_{r}}{d r}+\frac{\sigma_{r}-\sigma_{\theta}}{r}+\rho \cdot \omega^{2} \cdot r=0 & \sigma_{r}=A-\frac{B}{r^{2}}-\left(\frac{3+v}{8}\right) \cdot \rho \cdot \omega^{2} \cdot r^{2} \\
\omega_{Y}=\frac{1}{r_{e}} \cdot \sqrt{\frac{8 \cdot \sigma_{Y}}{(3+v) \cdot \rho}} & \sigma_{\theta}=A+\frac{B}{r^{2}}-\left(\frac{1+3 \cdot v}{8}\right) \cdot \rho \cdot \omega^{2} \cdot r^{2} \\
\omega_{Y}=\sqrt{\frac{4 \cdot \sigma_{Y}}{\rho \cdot\left[(3+v) \cdot r_{e}^{2}+(1-v) \cdot r_{i}^{2}\right]}} & \sigma_{r 1} \cdot z_{1}=\sigma_{r 2} \cdot z_{2} \quad F_{c}=m \cdot \omega^{2} \cdot r
\end{array}
$$

## Torsion of Non-circular sections

$$
T=\alpha b t^{2} \frac{G \theta}{L}=\beta b t^{3} \frac{G \theta}{L} \quad \tau_{\max }=\frac{G \theta t}{L} \quad T=\frac{1}{3} b t^{2} \frac{G \theta}{L}
$$

| $\mathbf{b} / \mathbf{t}$ | $\mathbf{1}$ | $\mathbf{1 . 5}$ | $\mathbf{2}$ | $\mathbf{2 . 5}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{6}$ | $\mathbf{1 0}$ | $\infty$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\alpha$ | 0.208 | 0.231 | 0.246 | 0.256 | 0.267 | 0.282 | 0.299 | 0.312 | 0.333 |
| $\beta$ | 0.141 | 0.196 | 0.229 | 0.249 | 0.263 | 0.281 | 0.299 | 0.312 | 0.333 |


| Shape | $\int_{\boldsymbol{A}} \frac{\boldsymbol{d} \boldsymbol{A}}{\boldsymbol{r}}$ |
| :---: | :---: |

