UNIVERSITY OF JOHANNESBURG

PROGRAM	: BACCALAUREUS ENGINEERING TECHNOLOGIAE *METALLURGY & CHEMICAL ENGINEERING*	
SUBJECT	: **PROCESS CONTROL 3B**	
CODE	: **PRCCHB3**	
DATE	: SUPLEMENTARY EXAM SSA 2019 JANUARY 2020	
DURATION	: (Y-PAPER) 09:00 - 12:00	
WEIGHT	: 40 : 60	
TOTAL MARKS	: 100	

EXAMINER	: MR MK KALENGA	5142
MODERATOR	: LM OMARI	
NUMBER OF PAGES	: 3 PAGES AND 2 ANNEXURES	

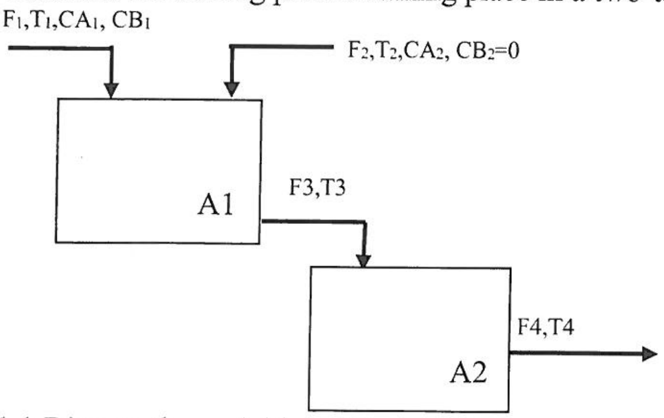
INSTRUCTIONS	: **QUESTION PAPERS MUST BE HANDED IN.**
REQUIREMENTS	: CALCULATORS ARE NOT REQUIRED

INSTRUCTIONS TO CANDIDATES: ***Question paper to be handed in***

PLEASE ANSWER ALL THE QUESTIONS.

QUESTION 1 (GA1)

Consider the mixing process taking place in a two-tank system as per figure below.



1.1 Discuss the variables of the system (5)
1.2 Establish the mass balance of the process (10)
1.3 Establish the energy balance of the system (10)

[25]

QUESTION 2

Conduct a qualitative analysis of the mathematical model established in question 1. Ensure that you have combined the energy balance and mass balance to generate only one mathematical model that would allow to oversee the process. State whether the process is stable or not.

[20]

QUESTION 3

Fund the solution of the following set of equations:

$dx_1/dt = 2x_1 + 3x_2 + 2 \quad with\ x_1(0) = 0$

$dx_1/dt = 2x_1 + 3x_2 + e^t \quad with\ x_1(0) = 0$

[20]

QUESTION 4

Consider the following second-order differential equation:

$$a_2 \frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = f(t)$$

where X(t) is considered to be in the form of a deviation variable with initial conditions

$$x(0) = \left(\frac{dx}{dt}\right)_{t=0} = 0$$

What would be the time function if:

1. $a_1{}^2 - 4a_2a_o = 0$ and $a_1=2$, $a_2=1$ and $a_o=1$
2. $a_1{}^2 - 4a_1a_o < 0$ and $a_1=2$, $a_2=2$ and $a_o=2$

.

[20]

QUESTION 5

Discuss the principal considerations that affect the scope of mathematical modeling of a metallurgical process.

[15]

QUESTION6

Calculate the time function of the following Laplace transform:

$\bar{X}(s) = 1/(s-1)^3(s+2)$

[15]

TOTAL : 100

Table 7.1
Laplace Transforms of Various Functions

Time function ($t \geq 0$)	Laplace transform
Unit impulse, $\delta(t_0)$	1
Unit pulse, $\delta_A(t)$	$\frac{1}{A}\frac{1-e^{-sA}}{s}$
Unit step	$\frac{1}{s}$
Ramp, $f(t) = t$	$\frac{1}{s^2}$
t^2	$\frac{2!}{s^3}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$

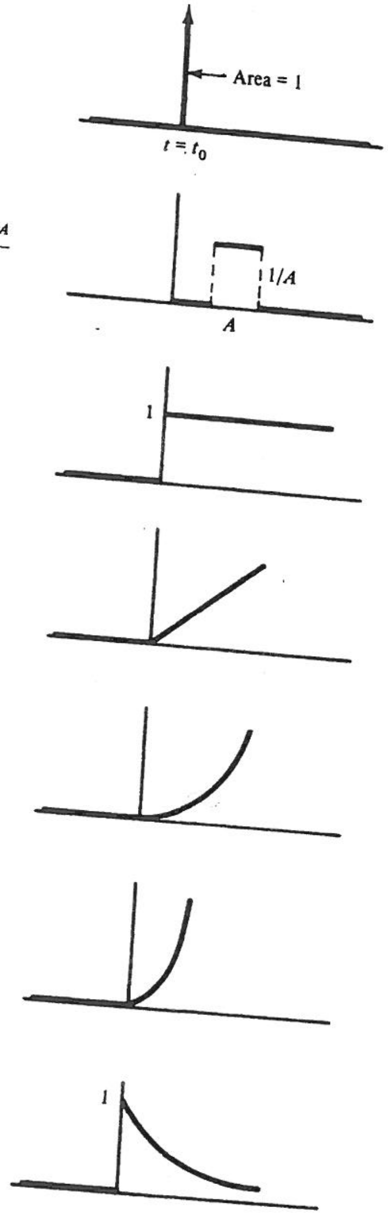
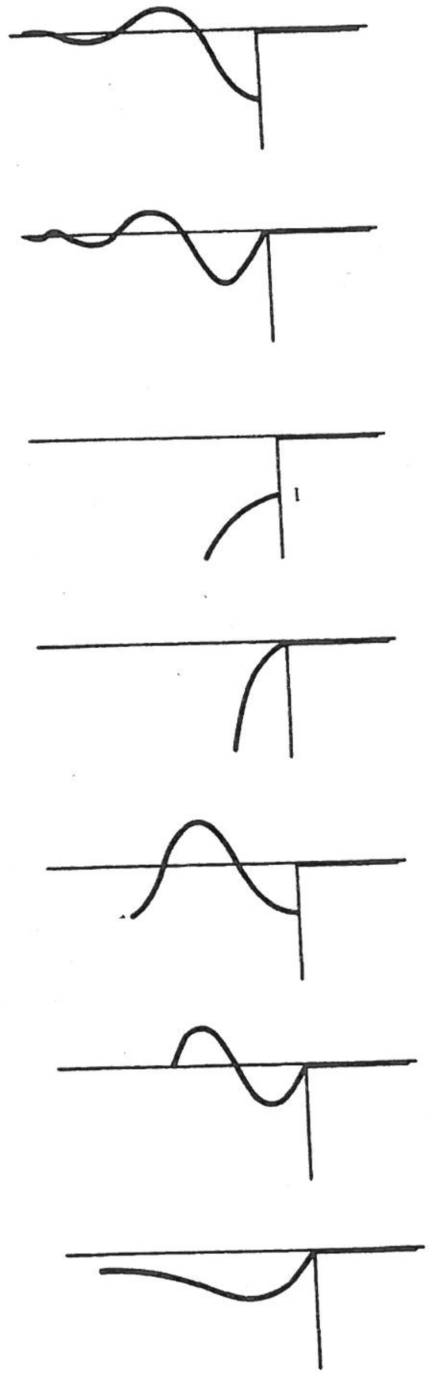


Table 7.1 (cont.)

Time function ($t \geq 0$)	Laplace transform
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega^2}$
$\cosh(\omega t)$	$\frac{s}{s^2-\omega^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{(s+a)}{(s+a)^2+\omega^2}$



transforms for typical functions such as Tables 7.1 and 8.1.

TABLE 8.1
INVERSE LAPLACE TRANSFORMS OF SELECTED EXPRESSIONS

	Laplace transform: $\bar{f}(s)$	Time function: $f(t)$
1.	$\dfrac{1}{(s+a)(s+b)}$	$\dfrac{e^{-at}-e^{-bt}}{b-a}$
2.	$\dfrac{1}{(s+a)(s+b)(s+c)}$	$\dfrac{e^{-at}}{(b-a)(c-a)}+\dfrac{e^{-bt}}{(c-b)(a-b)}+\dfrac{e^{-ct}}{(a-c)(b-c)}$
3.	$\dfrac{s+a}{(s+b)(s+c)}$	$\dfrac{1}{c-b}[(a-b)e^{-bt}-(a-c)e^{-ct}]$
4.	$\dfrac{a}{(s+b)^2}$	ate^{-bt}
5.	$\dfrac{a}{(s+b)^3}$	$\dfrac{a}{2}t^2e^{-bt}$
6.	$\dfrac{a}{(s+b)^{n+1}}$	$\dfrac{a}{n!}t^n e^{-bt}$
7.	$\dfrac{1}{s(as+1)}$	$1-e^{-t/a}$
8.	$\dfrac{1}{s(as+1)^2}$	$1-\dfrac{a+t}{a}e^{-t/a}$
9.	$\dfrac{\omega^2}{s(s^2+2\zeta\omega s+\omega^2)}$	$1+\dfrac{e^{-\zeta\omega t}}{\sqrt{1-\zeta^2}}\sin(\omega\sqrt{1-\zeta^2}\,t-\phi)$ where $\cos\phi=-\zeta$
10.	$\dfrac{s}{(1+as)(s^2+\omega^2)}$	$-\dfrac{1}{1+a^2\omega^2}e^{-t/a}+\dfrac{1}{\sqrt{1+a^2\omega^2}}\cos(\omega t-\phi)$ where $\phi=\tan^{-1}a\omega$
11.	$\dfrac{s}{(s^2+\omega^2)^2}$	$\dfrac{1}{2\omega}t\sin\omega t$
12.	$\dfrac{1}{(s+a)[(s+b)^2+\omega^2]}$	$\dfrac{e^{-at}}{(a-b)^2+\omega^2}+\dfrac{e^{-bt}\sin(\omega t-\phi)}{\omega[(a-b)^2+\omega^2]^{1/2}}$