UNIVERSITY OF JOHANNESBURG

PROGRAM	:	BACHELOR OF ENGINEERING TECHNOLOGY INDUSTRIAL ENGINEERING TECHNOLOGY
SUBJECT	:	**OPERATIONS RESEARCH**
CODE	:	**OPRMIB2**
DATE	:	YEAR END EXAMINATIONS 2019 12 NOVEMBER 2019
DURATION	:	12:30-15:30
WEIGHT	:	40 : 60
TOTAL MARKS	:	100

EXAMINER	:	MRS STEENKAMP
MODERATOR	:	MRS MAWANE
NUMBER OF PAGES	:	4 PAGES AND 2 ANNEXURES

INSTRUCTIONS	:	PLEASE ANSWER ALL THE QUESTIONS.
REQUIREMENTS	:	STUDENTS MAY USE CALCULATORS

QUESTION 1

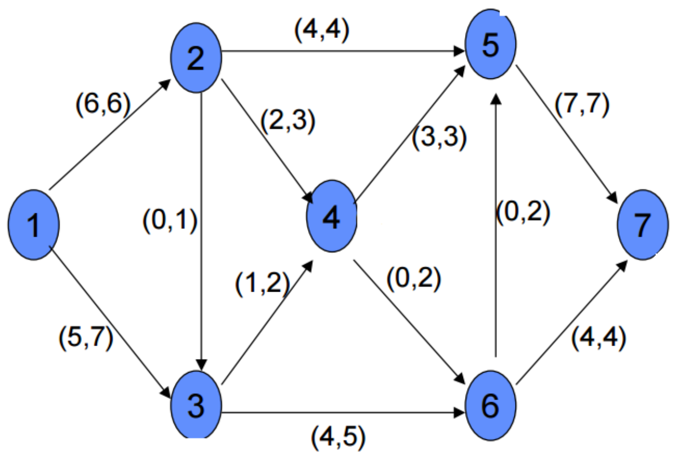
A company is considering producing some new Gameboy electronic games. Based on past records, management believes that there is a 70 percent chance that each of these will be successful and a 30 percent chance of failure. Market research may be used to revise these probabilities. In the past, the successful products were predicted to be successful based on market research 90 percent of the time. However, for products that failed, the market research predicted these would be successes 20 percent of the time. If market research is performed for a new product, what is the probability that the results indicate a successful market for the product and the product is actually not successful? **[8]**

QUESTION 2

Lerato is going to fly to London on March 5, 2020 and return home on March 20, 2020. It is now November 1; on November 1 she may buy a one way ticket for R4000.00 or a round trip ticket for R 7000.00. She could also wait for December 1 at which time a one-way ticket will be R5000 and a return ticket will be R8000. Between November 1 and December 1 it is possible that her sister, who works for the airline company, will be able to obtain a free one-way ticket for Lerato. The probability that her sister will obtain a free ticket is 0.3. If Lerato has bought a round trip ticket on November 1, she may return half of her round trip ticket to the airline. In this case her total cost will be R3500 plus a R250 penalty. Use a decision tree to minimise Lerato's cost and determine her expected cost of obtaining round trip transportation to London.

[25]

QUESTION 3



Use max flow technique to determine the max flow through the network **[10]**

QUESTION 4

The Ramaru oil company manufactures has 5000 barrels of oil 1 and 10 000 barrels of oil 2. The company sells two products petrol and heating oil. Both products are produced by combining oil 1 and oil 2. The quality level of each oil is as follows oil 1: 10 and oil 2: 5. Petrol must have an average quality level of at least 8, and heating oil must have an average quality level of 6. Demand for each product must be created by advertising. Each 10 Rands spent on advertising on petrol creates 5 barrels of demand and each 10 Rands spent on advertising of heating oil creates a 10 barrel demand. Petrol is sold for R2 300 and heating oil R1 500 per barrel. Formulate a LP and solve it graphically to help Ramaru to maximise profit.

[15]

QUESTION 5

L'oreal is optimising their distribution, they have factories in Johannesburg, Midrand and Centurion, they have distribution centers as indicated in the table below

5.1. Set up a transportation problem using the Northwest corner rule.

5.2. Use the stepping stone method to determine the next table to solve this problem (one iteration)

	Randburg	Menlyn	Bronberick	North Riding	Supply
Johannesburg	5	8	4	6	590
Midrand	7	4	8	8	710
Centurion	6	3	9	5	950
Demand	450	720	530	550	

[14]

QUESTION 6

The copy machine in an office is very unreliable. If it was working yesterday, there is an 80% chance it will work today. If it was not working yesterday, there is a 10% chance it will work today. If it is not working today, what is the probability that it will be working 2 days from now?

[8]

QUESTION 7

The public transport ridership in Cape Town during the summer months is believed to be linked to the number of tourists.

Year	No of Tourists (100 000)	No of people on buses (Ridership) (10 000)
1	12	20
2	10	17
3	11	18.5
4	14	25
5	18	40
6	15	27
7	13	22

7.1. Calculate the three year moving average (3)

7.2 Develop a regression relationship (3)

7.3. Calculate the MAD (4)

[10]

QUESTION 8

A marketing research firm would like to survey undergraduate and graduate college students. They would like to do the survey to determine whether they take out student loans for their education. There are different cost implications for the region of the country where the college is located and the type of degree. The survey cost table is provided below:

	Student type	
Region	Undergraduate	Graduate
East	R10	R15
Central	R12	R18
West	R15	R21

The requirements for the survey are as follows:

The survey must have at least 1500 students

At least 400 graduate students

At least 100 graduate students should be from the West

No more than 500 undergraduate students should be from the East

At least 75 graduate students should be from the Central region

At least 300 students should be from the West

8.1. Formulate an LP problem

[10]

TOTAL : 100

FULL MARKS : 100

Key Equations

(2-1) $0 \leq P(\text{event}) \leq 1$

A basic statement of probability.

(2-2) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Probability of the union of two events.

(2-3) $P(A|B) = \dfrac{P(AB)}{P(B)}$

Conditional probability.

(2-4) $P(AB) = P(A|B)P(B)$

Probability of the intersection of two events.

(2-5) $P(A|B) = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$

Bayes' Theorem in general form.

(2-6) $E(X) = \sum_{i=1}^{n} X_i P(X_i)$

An equation that computes the expected value (mean) of a discrete probability distribution.

(2-7) $\sigma^2 = \text{Variance} = \sum_{i=1}^{n} [X_i - E(X)]^2 P(X_i)$

An equation that computes the variance of a discrete probability distribution.

(2-8) $\sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2}$

An equation that computes the standard deviation from the variance.

(2-9) Probability of r successes in n trials $= \dfrac{n!}{r!(n-r)!} p^r q^{n-r}$

A formula that computes probabilities for the binomial probability distribution.

(2-10) Expected value (mean) $= np$

The expected value of the binomial distribution.

(2-11) Variance $= np(1-p)$

The variance of the binomial distribution.

(2-12) $f(X) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

The density function for the normal probability distribution.

(2-13) $Z = \dfrac{X - \mu}{\sigma}$

An equation that computes the number of standard deviations, Z, the point X is from the mean μ.

(2-14) $f(X) = \mu e^{-\mu x}$

The exponential distribution.

(2-15) Expected value $= \dfrac{1}{\mu}$

The expected value of an exponential distribution.

(2-16) Variance $= \dfrac{1}{\mu^2}$

The variance of an exponential distribution.

(2-17) $P(X \leq t) = 1 - e^{-\mu t}$

Formula to find the probability that an exponential random variable, X, is less than or equal to time t.

(2-18) $P(X) = \dfrac{\lambda^x e^{-\lambda}}{X!}$

The Poisson distribution.

(2-19) Expected value $= \lambda$

The mean of a Poisson distribution.

(2-20) Variance $= \lambda$

The variance of a Poisson distribution.

(3-1) $\text{EMV}(\text{alternative } i) = \sum X_i P(X_i)$

An equation that computes expected monetary value.

(3-2) EVwPI $= \sum$(Best payoff in state of nature i) $\times$ (Probability of state of nature i)

An equation that computes the expected value with perfect information.

(3-3) EVPI = EVwPI − Best EMV

An equation that computes the expected value of perfect information.

(3-4) EVSI = (EV with SI + cost) − (EV without SI)

An equation that computes the expected value (EV) of sample information (SI).

(3-5) Efficiency of sample information $= \dfrac{\text{EVSI}}{\text{EVPI}} 100\%$

An equation that compares sample information to perfect information.

(3-6) $P(A|B) = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$

Bayes' Theorem—the conditional probability of event A given that event B has occurred.

(3-7) Utility of other outcome $= (p)(1) + (1 - p)(0) = p$

An equation that determines the utility of an intermediate outcome.

(5-1) $\text{MAD} = \dfrac{\sum|\text{forecast errors}|}{n}$

A measure of overall forecast error called mean absolute deviation.

(5-2) $\text{Moving average} = \dfrac{\sum \text{demand in previous } n \text{ periods}}{n}$

An equation for computing a moving average forecast.

(5-3) Weighted moving average

$= \dfrac{\sum(\text{weight for period } n)(\text{demand in period } n)}{\sum \text{weights}}$

An equation for computing a weighted moving average forecast.

(5-4) New forecast = last period's forecast + α(last period's actual demand − last period's forecast)

An equation for computing an exponential smoothing forecast.

(5-5) $F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$

Equation 5-4 rewritten mathematically.

(5-6) $T_t = (1-\beta)T_{t-1} + \beta(F_t - F_{t-1})$

Trend component of an exponential smoothing model.

(5-7) $\hat{Y} = a + bX$

A least squares straight line used in trend projection and regression analysis forecasting.

(5-8) $b = \dfrac{\sum XY - n\bar{X}\bar{Y}}{\sum X^2 - n\bar{X}^2}$

An equation used to compute the slope, b, of a regression line.

(5-9) $a = \bar{Y} - b\bar{X}$

An equation used to compute the Y-intercept, a, of a regression line.

(5-10) $S_{Y,X} = \sqrt{\dfrac{\sum(Y - \hat{Y})^2}{n-2}}$

Standard error of the estimate.

(5-11) $S_{Y,X} = \sqrt{\dfrac{\sum Y^2 - a\sum Y - b\sum XY}{n-2}}$

Another way to express Equation 5-10.

(5-12) $r = \dfrac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$

Correlation coefficient.

(5-13) $\hat{Y} = a + b_1X_1 + b_2X_2$

The least squares line used in multiple regression.

(5-14) $\text{Tracking signal} = \dfrac{\text{RSFE}}{\text{MAD}} = \dfrac{\sum(\text{actual demand in period } i - \text{forecast demand in period } i)}{\text{MAD}}$

An equation for monitoring forecasts.

1) (New row numbers) = (numbers in old row) $- \left[\begin{pmatrix}\text{number above or below}\\ \text{pivot number}\end{pmatrix} \times \begin{pmatrix}\text{corresponding number}\\ \text{in newly replaced row}\end{pmatrix}\right]$

Formula for computing new values for nonpivot rows in the simplex tableau (step 4 of the simplex procedure).

10-1) $R_i + K_j = C_{ij}$

An equation used to compute the MODI cost values (R_i, K_j) for each column and row intersection for squares used in the solution.

(10-2) Improvement index $(I_{ij}) = C_{ij} - R_i - K_j$

The equation used to compute the improvement index for each unused square by the MODI method. If all improvement indices are greater than or equal to zero, an optimal solution has been reached.

(16-1) $\pi(i) = (\pi_1, \pi_2, \pi_3, \ldots, \pi_n)$

The vector of state probabilities for period i.

(16-2) $P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ \vdots & & & & \vdots \\ P_{m1} & P_{m2} & P_{m3} & & P_{mn} \end{bmatrix}$

The matrix of transition probabilities, that is, the probability of going from one state into another.

(16-3) $\pi(1) = \pi(0)P$

Formula for calculating the state 1 probabilities given state 0 data.

(16-4) $\pi(n+1) = \pi(n)P$

Formula for calculating the state probabilities for the period $n+1$ if we are in period n.

(16-5) $\pi(n) = \pi(0)P^n$

Formula for computing the state probabilities for period n if we are in period 0.

(16-6) $\pi = \pi P$ at equilibrium

The equilibrium state equation used to derive equilibrium probabilities.

(16-7) $P = \left[\begin{array}{c|c} I & 0 \\ \hline A & B \end{array}\right]$

The partition of the matrix of transition for absorbing state analysis.

(16-8) $F = (I - B)^{-1}$

The fundamental matrix, used in computing probabilities of ending up in an absorbing state.

(16-9) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \begin{bmatrix} \frac{d}{r} & \frac{-b}{r} \\ \frac{-c}{r} & \frac{a}{r} \end{bmatrix}$ where $r = ad - bc$

The inverse of a matrix with 2 rows and 2 columns.