University of Johannesburg
Department of Physics
PHYE0A2/PHYE2A2:
Static and Dynamic Electromagnetism
Exam (01 June 2019)
Examiner: Dr. R. Warmbier
Moderator: Dr. C.J. Sheppard
Time: 150 minutes

Student No.: ______________ **Surname, Initials:** ______________

Instructions:

- Enter your Student number, row & seat number above.
- Read all questions and instructions carefully. It is your responsibility to make sure that your paper has 17 pages (excluding the cover-sheet(s)).
- Answer the written questions on the question paper.

Written Questions

- **Draw** diagrams where appropriate. Marks are allocated for diagrams.
- **Show** all work, clearly and in order, if you want to get full credit. **Justify** the steps you take to ensure full marks. We reserve the right to take off marks if we cannot see how you arrived at your answer (even if your final answer is correct). Please keep your written answers **brief**; be clear and to the point. We reserve the right to take points off for rambling, incorrect or irrelevant statements.
- Do algebra with variables. Numerical values can be substituted at the end. Numerical work will only be evaluated at the last step.
- Underline or otherwise indicate your final answers.

Written Marks	
Q 1	/12
Q 2	/12
Q 3	/08
Q 4	/10
Q 5	/12
Q 6	/08
Q 7	/13
Q 8	/11
Tot.	/86

Mark Summary
Avail. marks: 86^{pts}
Full Marks: 80^{pts}

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12 pts

1. Consider a linear, isotropic and non-homegeneous dielectric material. The free and bound volume charge densities are represented by ρ_f and ρ_b, respectively.

(a) (6 pts) Starting from Gauss' law, show that $\rho_f = \nabla \cdot \vec{\mathbf{D}}$, where $\vec{\mathbf{D}}$ is the dielectric displacement vector.

(b) (2 pts) Is $\vec{\mathbf{D}}$ only dependent on the free volume charge density in the medium or also on other charge densities? If yes, which? Explain your answer. (not more than 2-3 sentences and/or equations)

(c) (4 pts) Show that $\rho_b = -\frac{\epsilon_r - 1}{\epsilon_r}\rho_f$. Hint: Show first that: $\vec{\mathbf{P}} = \frac{\epsilon_r - 1}{\epsilon_r}\vec{\mathbf{D}}$.

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12 pts

2. You are given two metallic spherical shells, where the smaller shell ($r_1 = 10\,\text{cm}$) is inside (at the center) the larger shell ($r_2 = 12\,\text{cm}$). The inner shell is held at a potential of $1\,\text{kV}$ with respect to the outer shell. A uniform dielectric medium, with a dielectric constant of $\varepsilon_r = 2.5$, completely fills the space between the two shells.

(a) (3 pts)

i) Show that the charge Q on the metal shells due to the potential difference is $Q = \pm 167\,\text{nC}$.

ii) Do the shells hold positive/negative charge? Why?

(b) (3 pts) Calculate $\vec{\mathbf{D}}$, $\vec{\mathbf{E}}$ and $\vec{\mathbf{P}}$ in the dielectrc medium.

(c) (2 pts) Calculate the free surface charge densities on each conductor.

(d) (3 pts) Calculate the bound charges in the dielectric.

(e) (1 pt) Calculate the capacitance of the system.

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8 pts

3. The magnitude of the current density $\vec{\mathbf{J}}$ in medium 1 is $J_1 = 50\,\mathrm{A\,m^{-2}}$. $\vec{\mathbf{J}}$ makes an angle of 30° with respect to the normal at the interface.
In medium 1: $\sigma_1 = 100\,\mathrm{S\,m^{-1}}$, $\epsilon_{r1} = 9.6$
In medium 2: $\sigma_2 = 10\,\mathrm{S\,m^{-1}}$, $\epsilon_{r2} = 4.0$

(a) (2 pts) Calculate the magnitude of the normal and tangential components of $\vec{\mathbf{J}}_1$.

(b) (3 pts)

i) Calculate the magnitude of the normal and tangential components of $\vec{\mathbf{J}}_2$ in medium 2.

ii) Calculate also the magnitude of $\vec{\mathbf{J}}_2$ and its angle to the normal.

(c) (3 pts) Calculate the surface charge density at the interface.

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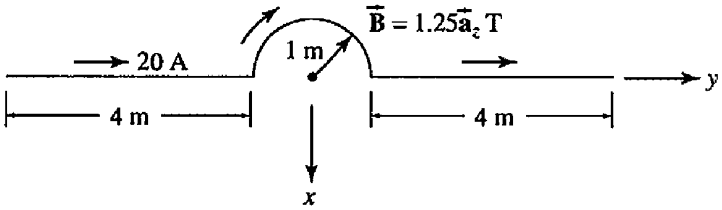
10 pts

4. A wire bent as shown in the figure, lies in the xy-plane and carries a current of $I = 20\,\mathrm{A}$. The magnetic field in the region is $\vec{\mathbf{B}} = 1.25\,\mathrm{T}\hat{\mathbf{z}}$. The straight parts of the wire are each 4 m long. The bend part of the wire has the shape of a semi-circle with radius 1 m. Determine the force experienced by the wire.
Hint: $\hat{\rho} = \hat{x}\cos\phi + \hat{y}\sin\phi$



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12 pts

5. A very long straight conductor located along the z-axis has a circular cross section with a radius of 10 cm. The conductor carries a current of 100 A in the positive z-direction.

(a) (3 pts) The non-uniform current density is described by the equation $\vec{\mathbf{J}}(\rho) = k\rho\hat{\mathbf{z}}$, where k is a constant and ρ is the radial coordinate. Show that for the above mentioned current the constant k will be $k \approx 47\,750\,\mathrm{A\,m^{-3}}$.

(b) (5 pts) Using the current density given in (a), find the magnetic field intensity $\vec{\mathbf{H}}$ inside the conductor.

(c) (2 pts) Calculate the magnetic field intensity $\vec{\mathbf{H}}$ outside the conductor.

(d) (2 pts) Sketch the field intensity $\vec{\mathbf{H}}$ as a function of the radial coordinate ρ. What (and where) is the maximum of $\vec{\mathbf{H}}$?

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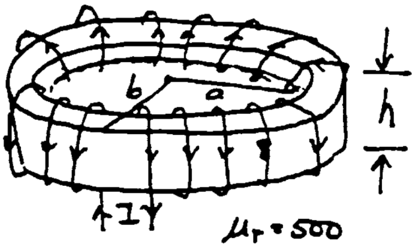
8 pts

6.



A toroid of square cross section is wound with $N = 200$ turns, as shown in the figure. The inner and outer radii of the toroid are $a = 20\,\text{cm}$ and $b = 25\,\text{cm}$, respectivally. Its height is $h = 5\,\text{cm}$. The relative permeability of the magnetic material is $\mu_r = 500$.

(a) (3 pts) Show that the magnetic flux density $\vec{\mathbf{B}}$ generated by the coil is given by $\vec{\mathbf{B}}(\rho) = \frac{\mu I N}{2\pi\rho}\hat{\phi}$ for $a \le \rho \le b$.

(b) (3 pts) Compute the self-inductance of the toroid.

(c) (2 pts) If the current in the coil varies as $I(t) = 2\sin(314t)$ A, determine the induced voltage V_i in the coil.

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13 pts

7. The electric field in a dielectric medium is given by $\vec{\mathbf{E}} = E_0 \cos(\omega t - \beta z)\hat{\mathbf{x}}\ \mathrm{V\,m^{-1}}$.

(a) (4 pts) Compute the magnetic flux density $\vec{\mathbf{B}}$ from the knowledge of the electric field, and show that it is given by $\vec{\mathbf{B}} = \frac{E_0\beta}{\omega}\cos(\omega t - \beta z)\hat{\mathbf{y}}$.

(b) (3 pts) Show that electric energy density is equal to the magnetic energy density, if $\beta^2 = \omega^2\mu\epsilon$.

(c) (3 pts)

i) Compute the Poynting vector.

ii) What is the physical meaning of the Poynting vector?

(d) (3 pts) Compute the average power density.

Hint: $\cos^2(a+b) = \frac{1}{2}\cos(2a+2b) + \frac{1}{2}$

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11 pts

8. Consider a linearly polarized electromagnetic wave traveling in the positive direction of the z-axis for which the electric field $\vec{\mathbf{E}}$ is $\vec{\mathbf{E}} = \vec{\mathbf{E}}_0 e^{i(\omega t - kz)}$. $\vec{\mathbf{E}}_0$ does not depend on time or position.

(a) (7 pts) Derive the wave equation $\nabla^2 \vec{\mathbf{E}} = \frac{1}{v^2} \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$ from the E-wave form assuming zero attenuation.

(b) (4 pts) Show that for the traveling wave in a vacuum it is a transverse wave.

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— CONTINUATION SPACE —

— END OF PAPER —