UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

PHYSICS

PHY002A/PHY00A2

EXAMINATION
JULY 2019

EXAMINER: **Prof CA Engelbrecht**

MODERATOR: Prof S Karataglidis

TIME: 3 HOURS **MARKS: 150**

This paper consists of 7 pages, including this cover page.

Please read the following instructions carefully:

1. ANSWER ANY FIVE QUESTIONS

2. INFORMATION THAT YOU MAY USE APPEARS AT THE END OF THE PAPER

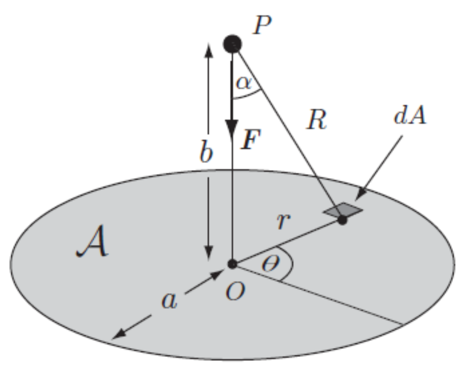
3. No programmable calculators are allowed.

Question 1: [30]

(a) Start from first principles and derive the following expression for the gravitational force experienced by a mass m located at the point P - as indicated on the adjoining diagram:

$$F = \frac{2GMm}{a^2}\left(1 - \frac{b}{(a^2+b^2)^{1/2}}\right)$$

(where M is the total mass of the thin, uniform disk, a is the radius of the disk and b the vertical distance of point P above the centre of the disk).



(11)

(b) Derive the following formula for the vertical speed of a projectile moving in a resistive fluid:

$$v_z = \frac{g}{K}e^{-Kt} + u\sin\alpha(e^{-Kt}) - \frac{g}{K}$$

Note: Clearly state which assumption(s) you are making and explain the meaning of each of the symbols appearing in this equation in the course of your derivation.

(12)

(c) Define the function $H(q, p, t)$ as the Legendre transform of the Lagrangian function L of a system. Use the definition of generalised momentum as a derivative of L to derive the Hamiltonian equations of motion:

$$\dot{p}_j = -\frac{\partial H}{\partial q_j} \qquad \text{and} \qquad \dot{q}_j = \frac{\partial H}{\partial p_j}$$ (7)

Question 2: [30]

(a) Consider an asteroid moving in a straight line, approaching the Sun from a far distance, with impact parameter p and initial speed u. The asteroid is deflected from its original path by the gravitational pull of the Sun. Draw an appropriate diagram of the deflected path, including a clear indication of the deflection angle, then use the appropriate L – and E-formulae to derive the following equation for the deflection angle:

$$\tan(\beta / 2) = \frac{M_{Sun} G}{p u^2} \tag{11}$$

(b) Derive an equation describing the displacement $x(t)$ of the damped linear oscillator which is driven by the periodic force:

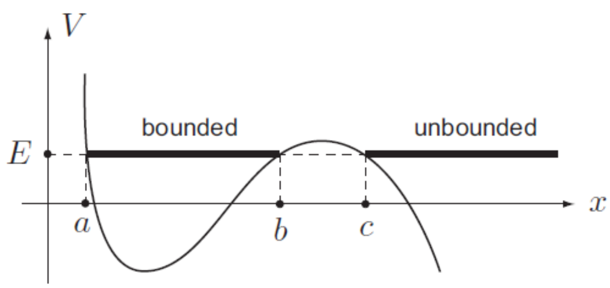
$$F(t) = \begin{cases} F_0 \text{ if } & 0 < t \leq \pi \\ -F_0 \text{ if } & \pi < t \leq 2\pi \end{cases}$$

(13)

(c) Consider a particle with total energy E, subject to the potential function depicted in the accompanying figure.

i) Use the principle of energy conservation to analyse the general motion of the particle.

ii) Assume that the particle is confined to the interval $[a, b]$ on the x axis. Derive an integral expression for the oscillation period of the particle.



(6)

Question 3: [30]

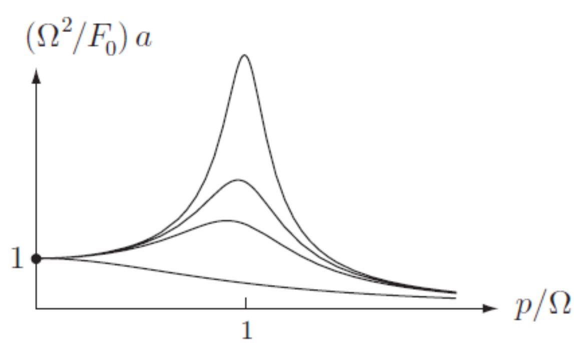
(a) i) Discuss the cases of underdamped and overdamped oscillations, respectively, of a free (undriven) one-dimensional harmonic oscillator. Clearly explain which assumptions you are making and include appropriate mathematical derivations in your discussion, to make the mathematical basis for the difference between the 2 cases clear.

(7)

ii) Use your discussion in i) to obtain an equation describing the motion $x(t)$ for the *under*damped case.

(7)

iii) Interpret the following graph: (4)

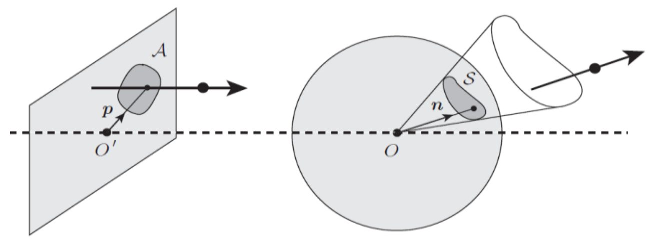


(b) Explain what is meant, respectively, by the '*space-like separation*' and '*time-like separation*' of two events. (3)

(c) Solve Hamilton's equations of motion for a particle orbiting in a plane, under the influence of an attractive, inverse-square central force. (9)

Question 4: [30]

(a) Consider the equality of the incoming and outgoing flux of a *beam* of particles crossing through an initial area *A* (on the left of the diagram shown below) and scattered through an exit area *S* (on the right of the diagram shown below)- by a repulsive central force:



as expressed in the equation $\int_S \sigma(\vec{n})dS = A$. Apply this equation to the case of Rutherford scattering and derive the Rutherford scattering cross section formula:

$$\sigma(\theta) = \frac{q^2Q^2}{16E^2}\left(\frac{1}{\sin^4(\theta/2)}\right)$$ (16)

(b) Show that the kinetic energy of a system of *N* particles, described by the generalized coordinates $\{q_1, q_2, ..., q_n\}$, can be written in the form:

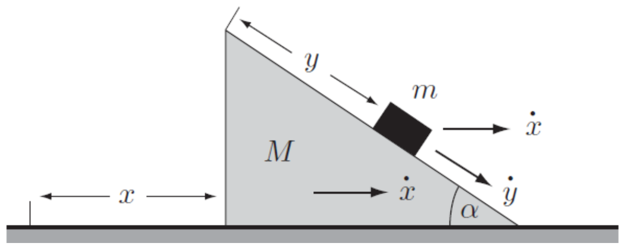
$$T = \sum_{j=1}^{n}\sum_{k=1}^{n} a_{jk}(\vec{q})\,\dot{q}_j\dot{q}_k\,, \text{ where } \quad a_{jk}(\vec{q}) = \tfrac{1}{2}\sum_{i=1}^{N} m_i\left(\frac{\partial \vec{r}_i}{\partial q_j}\right)\cdot\left(\frac{\partial \vec{r}_i}{\partial q_k}\right).$$ (6)

(c) Apply the Lorentz transformations to the concept of a '*sphere of light*' to introduce the concept of a '*Lorentz invariant'* quantity. Define the *space-time interval* as part of your answer. (6)

(d) Explain what is meant by the concept of *phase space* and how this concept is useful. (2)

Question 5: [30]

(a) Consider a block sliding without friction down the diagonal side of a wedge-shaped base, which can move freely (without friction) on a vertical plane, as shown in the accompanying diagram:



i) Obtain the equations of motion of the system by solving Lagrange's equations of motion . (12)

ii) Obtain the equations of motion of the system by solving Hamilton's equations of motion. (6)

(b) Explain what we learn about a system by identifying its *cyclic coordinates*. Clearly motivate your answer. (2)

(c) Make appropriate assumptions and derive the following form of the *Lorentz transformation equations* for space and time:

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}}; \quad t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} \qquad (10)$$

Question 6: [30]

(a) Consider a mechanical system which is holonomic and in which the constraint forces do no virtual work (i.e. a *standard system*): Assume that the position vectors of all the particles in the system can be fully described by a set of n *generalized co-ordinates*, expressed as $q_1, q_2, \ldots\ldots, q_n$ or just $\{q_j\}$ for short.

i) Derive an expression for the general velocity vector $\vec{v}_i$ of a particle in the system, expressed in terms of the *generalized co-ordinates* and *generalised velocities*. (3)

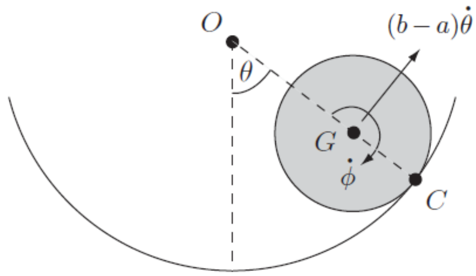
ii) Derive the following form of Lagrange's equations of motion, clearly indicating the appropriate assumptions that you make:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = \sum_{i=1}^{N} m_i \dot{\vec{v}}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j} \quad \text{for} \quad j = 1, 2, \ldots, n \qquad (13)$$

(b) A homogeneous cylinder with radius a is rolling - without slipping – along the inside surface of a pipe with a spherical circumference and radius b, as shown in the accompanying diagram. The axis of symmetry of the rolling cylinder is shown by *G.*

i) Identify and write down all the generalised coordinates that are necessary to describe the motion of the pipe;

ii) Then, use Lagrange's equations of motion to derive Lagrange's equations of motion for each of the generalized coordinates:



(9)

(c) Consider the motion of a particle with mass m in a *central* force field, expressed in *plane polar co-ordinates* (see the equations appearing at the end of the paper). Explain why the angular momentum of such a particle will be conserved, using appropriate mathematics to support your explanation. (5)

INFORMATION YOU MAY USE:

velocities and accelerations in plane polar co-ordinates:

$$\dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

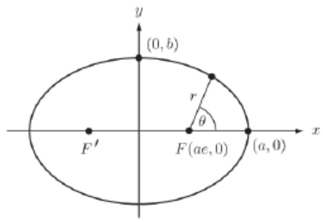
The E-formula

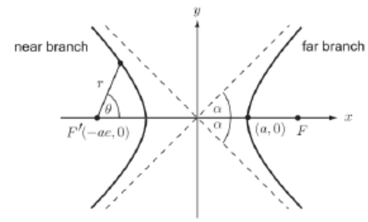
Ellipse:	$E < 0$	$E = -\frac{\gamma}{2a}$
Parabola:	$E = 0$	
Hyperbola:	$E > 0$	$E = +\frac{\gamma}{2a}$

The L-formula

$$L^2 = \gamma b^2 / a$$

$$Q_j = \sum_i \boldsymbol{F}_i^S \cdot \frac{\partial \boldsymbol{r}_i}{\partial q_j} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0 \qquad (1 \le j \le n)$$





$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad\qquad \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$e^2=1-\frac{b^2}{a^2} \qquad\qquad e^2=1+\frac{b^2}{a^2}$$

$$\frac{1}{r}=\frac{a}{b^2}(e\cos\theta+1) \qquad\qquad \frac{1}{r}=\frac{a}{b^2}(e\cos\theta\pm 1)$$

$$\tan\alpha=b/a$$

END OF EXAM