PROGRAM	:	BACHELOR OF ENGINEERING *MECHANICAL ENGINEERING*
SUBJECT	:	**THEORY OF MACHINES 3B**
CODE	:	**MKE3B21 / MKEMCB3**
DATE	:	SUPPLEMENTARY EXAMINATION JANUARY 2020
DURATION	:	(1-PAPER) 3 HOURS
WEIGHT	:	50 : 50
TOTAL MARKS	:	111
FULL MARKS	:	100

EXAMINER	:	Dr CR BESTER
MODERATOR	:	Prof AL NEL
NUMBER OF PAGES	:	2 PAGES INSTRUCTIONS 5 PAGES QUESTIONS 3 PAGES ANNEXURE (FORMULAE)

INSTRUCTIONS	:	SEE NEXT PAGE
REQUIREMENTS	:	NONE

<u>INSTRUCTIONS TO CANDIDATES</u>:

- FORMULA SHEETS ARE ATTACHED
- NO BOOKS, LECTURE NOTES, STUDY-, HOMEWORK- OR TUTORIAL MATERIAL ALLOWED
- UJ APPROVED CALCULATORS ALLOWED
- ANSWER ALL 5 QUESTIONS IN ENGLISH
- SMOKING IS PROHIBITED DURING THE DURATION OF THE EXAM

Question 1 **(36 marks)**

Figure 1 shows a rigid massless lever connected to a mass *m*, spring with a stiffness *k* and damper with a viscous damping coefficient *c*. The total length of the lever between the spring and mass is *l*. The lever pivots about point *A*. The angular displacement of the lever and vertical displacement of the mass are θ and *x* respectively. The horizontal distance between the spring and pivot is *a* and that between the pivot and damper is *b*.

Derive the equations of motion of the system for both rotational- and translational coordinate systems, with θ and *x* as the displacement coordinates. Also calculate the undamped- and damped natural frequencies for both coordinate systems, for the following parameters:

k: 5 000 N/m
m: 20 kg
c: 50 Ns/m

a: 100 mm
b: 200 mm
l: 400 mm

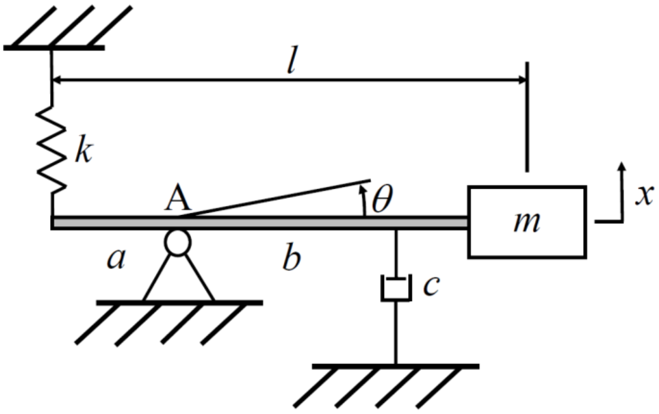

Figure 1: Rigid massless lever

Question 2 **(26 marks)**

Equation (1) gives an assumed deflection shape ψ_1 of the fundamental bending mode of a guided-guided beam in terms of *x*/*l*:

$$\psi_1 = 2\left(\frac{x}{l}\right)^5 - 5\left(\frac{x}{l}\right)^4 + 5\left(\frac{x}{l}\right)^2 - 1 \cdots\cdots(1)$$

The modal stiffness and mass are given by:

$$k_1 = \int_0^l EI\left(\frac{d^2\psi_1}{dx^2}\right)^2 dx \cdots\cdots(2a)$$

$$m_1 = \int_0^l \rho A\ \psi_1^2 dx \cdots\cdots(2b)$$

Determine the eigenvalue of the fundamental bending mode of the beam.

Question 3 **(11 marks)**

A piston-powered, propeller-driven aircraft makes a vertical circular loop with a radius of 500 m (see Figure 2). The engine and propeller are directly-coupled and both run anti-clockwise at 2 200 RPM, looking from aft. The tangential speed of the aircraft in the loop is constant at 200 km/h. The combined moment of inertia of the engine and propeller is 17 kgm^2. Determine the magnitude and direction of the gyroscopic moment of the engine and propeller acting on the aircraft, as well as the response of the aircraft to the gyroscopic effect of the engine and propeller.

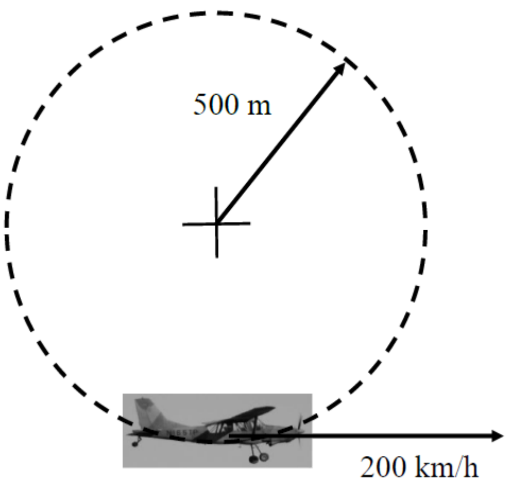

Figure 2: Aircraft making a vertical loop

Question 4 **(20 marks)**

Consider the schematic of a velometer shown in Figure 3. A velometer estimates the velocity V_b of the base from the relative velocity V_r, which is the difference between the velocity V of the mass and velocity V_b of the base.

The ratio of relative velocity to true base velocity, for harmonic base motion, is given by:

$$\left|\frac{V_r}{V_b}\right| = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \qquad (1)$$

where the symbols denote the following:

- r: Frequency ratio, i.e. ω/ω_n
- ζ: Dimensionless damping factor
- V_r: Relative velocity, i.e. difference between the velocity of the mass V and that of the base V_b
- V_b: True velocity, i.e. velocity of the base V_b

Figure 4 [Figliola & Beasley, 2011]* shows a graph of $|V_r/V_b|$ against r and dimensionless damping factor.

For any given damping factor $\zeta < 1/\sqrt{2}$, the ratio $|V_r/V_b|$ reaches a peak. For that range of damping factors, derive an equation for:

(i) the frequency ratio r where $|V_r/V_b|$ reaches a peak
(ii) the peak ratio $|V_r/V_b|$

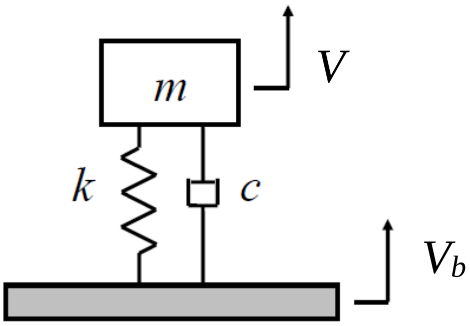

Figure 3: Velometer

* Figliola, R.S., & Beasley, D.E., "Theory and Design for Mechanical Measurements," 5th Edition, John Wiley & Sons, 2011, p. 513, Figure 12.9

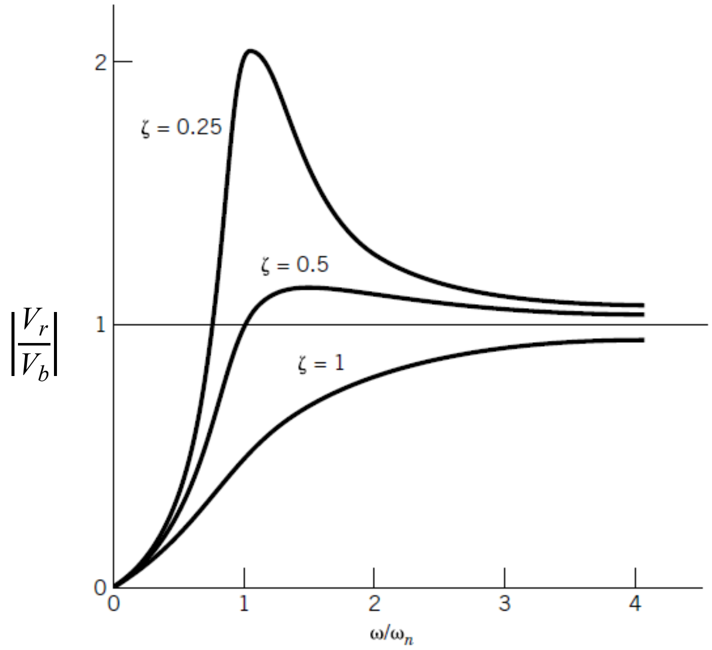

Figure 4: $|V_r/V_b|$ against *r* and damping factor [Figliola & Beasley, 2011]*

Question 5 **(18 marks)**

An electrohydraulic servomotor is schematically shown in Figure 5*. The servomotor consists of an electrohydraulic servovalve, actuator, load and rigid massless lever connecting the valve- and actuator rods.

The load mass is denoted by m. The input displacement of the lever is x, which is applied at the left end of the lever. The displacement of the valve rod is z and that of the actuator and load is y.

Consider the special case where $a = b$. For x as the input to the servomotor and y as the output, the Laplace-domain transfer function $Y(s)/X(s)$ may be given by:

$$\frac{Y(s)}{X(s)} = \frac{\dfrac{K_1 C_1 A_p}{2K_2 m}}{s^2 + \dfrac{A_p^2}{K_2 m}s + \dfrac{K_1 C_1 A_p}{2K_2 m}}$$

where

K_1 is the no-load flow per unit current
K_2 is the flow per unit load pressure at a constant current
C_1 is the servovalve spool current per unit displacement
A_p is the piston cross-sectional area

Determine the time-domain response $y(t)$ of the servomotor if x is a unit step displacement applied at time $t = 0$.

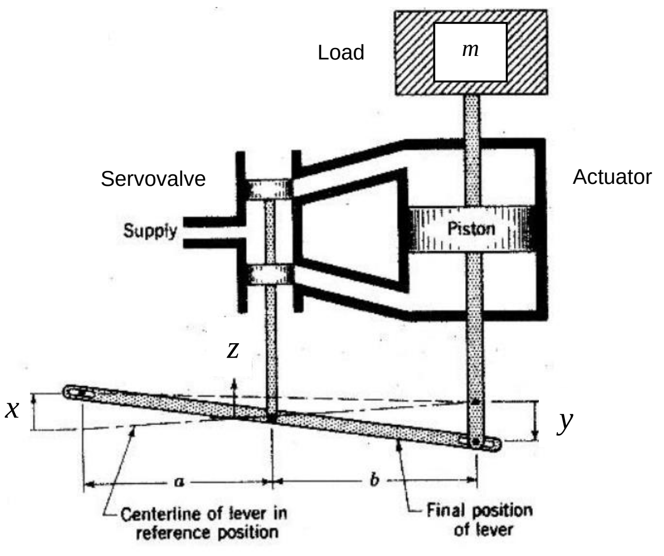

Figure 5: Electrohydraulic servomotor*

* Raven, F.H., "Automatic Control Engineering," 3rd Edition, McGraw-Hill Kogakusha, Tokyo, 1978, p. 57, Fig. 3.7, altered

Various formulae

$$\frac{d}{d\theta}(\sec\theta) = \sec\theta\tan\theta$$

$$P = mf$$

$$Fa = \{mf + S(x+y)\}(R+r_0)\sec\theta\tan\theta$$

$$Fa = F(\rho - R)\sin\theta$$

$$F_r = mr\Omega^2$$

$$F_{ix} = m_i r_i \Omega^2 \cos\theta_i$$

$$\sum \overline{F} + \overline{R} = \overline{0}$$

$$\overline{R} = -\sum \overline{F}$$

$$F_r = m\frac{v_\theta^2}{r} = m\frac{(r\Omega)^2}{r} = mr\Omega^2$$

$$n = r/a$$

$$\Omega = \dot{\theta}$$

$$Bb = Rr$$

$$x = (R+r_0)(\sec\theta - 1)$$

$$\dot{x} = \frac{dx}{dt} = \omega(R+r_0)\sec\theta\tan\theta$$

$$\ddot{x} = \omega^2(R+r_0)(2\sec^3\theta - \sec\theta)$$

$$x = (\rho - R)(1-\cos\theta)$$

$$\dot{x} = \omega(\rho - R)\sin\theta$$

$$\ddot{x} = \omega^2(\rho - R)\cos\theta$$

$$x = d(1-\cos\theta)$$

$$\dot{x} = \omega d\sin\theta$$

$$a = (R+r_0)\tan\theta$$

$$a = d\sin\theta$$

$$l = d - R + r$$

$$I_Q = I_G + \underbrace{md^2}_{\text{Steiner term}}$$

$$T_A/T_P = D_A/D_P$$

$$\frac{d}{d\theta}(\sec\theta\tan\theta) = 2\sec^3\theta - \sec\theta$$

$$F\cos\theta = mf + S(x+y)$$

$$F = mf + S(x+y)$$

$$Fa = \{mf + S(x+y)\}(\rho - R)\sin\theta$$

$$\ddot{x} \approx a\Omega^2\left(\cos\theta + \frac{\cos 2\theta}{l/r}\right)$$

$$F_{iy} = m_i r_i \Omega^2 \sin\theta_i$$

$$\overline{R} = -\sum \overline{F} = -\overline{F_1} - \overline{F_2}$$

$$\overline{R} = R_x\overline{i} + R_y\overline{j}$$

$$\overline{F_i} = F_{ix}\overline{i} + F_{iy}\overline{j}$$

$$\frac{d^2x}{dt^2} \approx a\Omega^2\left(\cos\theta + \frac{\cos 2\theta}{r/a}\right) = a\Omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$P = R\frac{d^2x}{dt^2} \approx R\omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{l/r}\right) = \underbrace{R\omega^2 r\cos\theta}_{\text{primary inertia force}} + \underbrace{\frac{R\omega^2 r^2}{l}\cos 2\theta}_{\text{secondary inertia force}}$$

$$\theta = \frac{\text{total rotation per cycle}}{\text{number of cylinders}}$$

$$\tan\beta = \frac{|\overline{AB}|}{|\overline{OQ}|} = \frac{d\sin\alpha}{R+r_0}$$

$$\ddot{x} = f = -\omega^2 d\left(1 + \frac{1}{n}\right)$$

$$\rho = \frac{R^2 - r^2 + d^2 - 2Rd\cos\alpha}{2(R - r - d\cos\alpha)}$$

$$\sin\psi = \frac{d\sin\alpha}{\rho - r}$$

$$x = (d\cos\phi + r) - R$$

$$\dot{x} = -\omega d\sin\phi$$

$$\ddot{x} = -\omega^2 d\cos\phi$$

$$\ddot{x} = \omega^2 d\cos\theta$$

$$a = (\rho - R)\sin\theta$$

$$Fa = (mf + S(x+y))d\sin\theta$$

$$T = Pa = \frac{m\omega^2}{2}(\rho - R)^2\sin 2\theta$$

$$D_P = (D_A - D_S)/2$$

$$T_A/T_S = D_A/D_S$$

Various formulae (continued)

$$\bar{\omega} = \frac{\omega_1 + \omega_2}{2}$$

$$\Delta E = \int T d\theta$$

$$\Delta E_{\max} = \frac{1}{2} I\left(\omega_1^2 - \omega_2^2\right)$$

$$\Delta E_{\max} = I\bar{\omega}\left(\omega_1 - \omega_2\right)$$

$$\sin(2\theta - \phi) = \sin 2\theta \cos\phi - \cos 2\theta \sin\phi$$

$$\omega = 2\pi N/60$$

$$\frac{a}{\sin a} = \frac{b}{\sin b} = \frac{c}{\sin c}$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\omega_n = \sqrt{k/m}$$

$$f = \omega/2\pi$$

$$\text{K.E.} = \frac{1}{2}J\omega^2$$

$$\text{K.E.} = \frac{1}{2}mv^2$$

$$m = \int_0^l \rho A\, \psi^2\, dx$$

$$k = \int_0^l EI \left(\frac{d^2\psi}{dx^2}\right)^2 dx$$

$$J_{eq}\ddot{\theta} + k_{\theta eq}\theta = 0$$

$$\text{P.E.} = mgh$$

$$\omega_n = \sqrt{k_\theta/J}$$

$$v = r\omega$$

$$\text{P.E.} = \frac{1}{2}kx^2$$

$$m\ddot{y} + c\dot{y} + ky = 0$$

$$m\ddot{y} + ky = 0$$

$$\zeta = c/(2m\omega_n) = c/\left(2\sqrt{km}\right)$$

$$\ddot{\theta} + \omega_n^2\,\theta = 0$$

$$\ddot{y} + \omega_n^2 y = 0$$

$$I = mk^2$$

$$M = I\omega\Omega$$

$$r = \omega/\omega_n$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_2 N_2 = -t_1 N_1$$

$$T_2 N_2 = T_1 N_1$$

$$\Omega = V/r$$

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

Table of useful Laplace-transforms

Function $f(t)$	**Laplace-transform** $F(s) = \mathcal{L}\{f(t)\}$
$y(t)$	$Y(s)$
$\dot{y}(t) = \frac{dy(t)}{dt}$	$sY(s) - y_0$
$\ddot{y}(t) = \frac{d\dot{y}(t)}{dt} = \frac{d^2y(t)}{dt^2}$	$s^2Y(s) - sy_0 - \dot{y}_0$
$f(t) = 0$ for $t \neq 0$; $\int_{-\infty}^{\infty} f(t)dt = 1$	1
$f(t) = 1$	$\frac{1}{s}$
$f(t) = t$	$\frac{1}{s^2}$
$f(t) = t^2$	$\frac{2}{s^3}$
$e^{at}y(t)$	$Y(s-a)$
$e^{-\zeta\omega_n t}$	$\frac{1}{s+\zeta\omega_n}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-\zeta\omega_n t}\sin\omega_d t$	$\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}$
$e^{-\zeta\omega_n t}\cos\omega_d t$	$\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$
$ty(t)$	$-\frac{dY(s)}{ds}$
$t\sin\omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$