



**FACULTY OF SCIENCE**

**DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS**  
**BACCALAUREUS OPTOMETRIAE**

**MODULE: MATHEMATICS 1A**  
**MAT01A1**  
**CAMPUS: DFC**  
**ASSESSMENT: JUNE EXAMINATION**

**DATE: 31 MAY 2019**

**TIME: 14:30 – 16:00**

**ASSESSOR:**

**MR. IK LETLHAGE**

**MODERATOR:**

**MR. EZ MORAPELI**

**DURATION: 2 HOURS**

**MARKS: 70**

<b>MARKS OBTAINED</b>	<div><hr/><b>70</b></div>	<b>%</b>
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**INITIALS AND SURNAME:** \_\_\_\_\_

**STUDENT NUMBER:** \_\_\_\_\_

**CONTACT NUMBER:** \_\_\_\_\_

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**THIS PAPER HAS 13 PAGES (INCLUDING THE COVER PAGE). ENSURE THAT YOU HAVE ALL THE PAGES.**

**INSTRUCTIONS: ANSWER ALL QUESTIONS IN THE SPACES PROVIDED.**  
**USE ONLY A PEN FOR WRITING AND DRAWING (BLACK OR BLUE).**  
**USE THE BLANK PAGES FOR ROUGH WORK AND MARK IT AS SUCH.**  
**SHOULD YOU REQUIRE MORE SPACE FOR YOUR ANSWERS, USE THE BLANK PAGES. CLEARLY INDICATE THE RELEVANT QUESTION.**  
**CALCULATORS ARE NOT ALLOWED.**

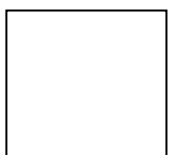
**QUESTION 1 [4]**

Solve the given inequality and represent the solution in interval form.

$$|4x+3| \geq 5$$

**QUESTION 2 [3]**

Solve the equation for  $x \in [0, 2\pi]$ :  $3 \tan \frac{x}{2} + 3 = 0$ .



**QUESTION 3 [3]**

Find the limit  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left[ \left( \frac{i}{n} \right)^2 - 1 \right]$

**QUESTION 4 [4]**

Find all the cube roots of  $-64i$ , and then give the answers in the form  $a + bi$



**QUESTION 5 [2]**

Suppose that it is known that  $B \rightarrow \neg A$  is true,  $\neg B \rightarrow \neg C$  is true and  $C$  is true. Determine the truth value of  $A$ . **Show how you reached your conclusion.**

**QUESTION 6 [4]**

Consider the statement: If  $n^2$  is even, then  $n$  is even.

a) Write the contrapositive of this statement. (1)

b) Use proof by contraposition to prove the given statement. (3)

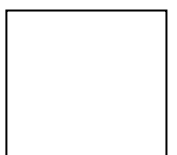


**QUESTION 7 [2]**

Let  $f(x) = \log_3 x - 5$ . Find the inverse of  $f : f^{-1}$ .

**QUESTION 8 [4]**

Find all the horizontal and vertical asymptotes of  $f(x) = \frac{5x^2 + 3x - 8}{x^2 - 2x + 1}$



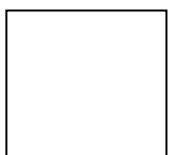
**QUESTION 9 [3]**

Consider the function  $f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$ ,  $c$  a constant.

Find the value of  $c$  that will make  $f$  continuous everywhere.

**QUESTION 10 [3]**

Use the Squeeze Theorem to find  $\lim_{x \rightarrow 0} x^2 \cos \frac{1}{x}$ .

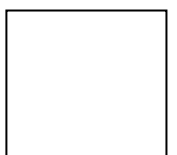


**QUESTION 11 [5]**

Let  $f(x) = \frac{1}{x^2}$ .

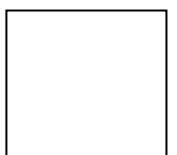
a) Use the definition of the derivative of a function to find  $f'(2)$ . (3)

b) Find the equation of the tangent to the curve  $y = \frac{1}{x^2}$  at the point  $\left(2; \frac{1}{4}\right)$ . (2)



**QUESTION 12 [4]**

Prove that  $\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$ ,  $\forall x \in \mathbb{R}$ .



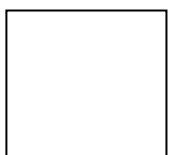


**QUESTION 13 [11]**

Find the indicated derivative, in its simplest form

a)  $\frac{dy}{dx}$  if  $y = \frac{x^x \tan x}{e^{x^2}}$  (4)

b)  $\frac{dy}{dx}$ , using implicit differentiation, if  $y^2 + x \ln y = 5x^3$  (3)



c)  $\frac{d^2 y}{dx^2}$  if  $y = x \sec^{-1} x$  (4)

**QUESTION 14 [3]**

Use l'Hôspital's Rule to find  $\lim_{x \rightarrow 0^+} x \ln x$



**QUESTION 15 [4]**

Find  $f$ , if  $f''(x) = x^3 + \sinh x$ ,  $f'(0) = 1$ ,  $f(2) = 2.6$ .

**QUESTION 16 [1]**

Use the Fundamental Theorem of Calculus to find  $\frac{d}{dx} \int_0^x e^{-t^2} dt$



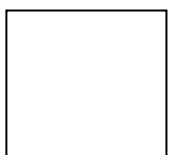
**QUESTION 17 [10]**

Evaluate the following integrals. Show all the steps.

a)  $\int_0^{\ln 3} e^{-x} \cosh 2x dx$  (4)

b)  $\int \frac{5x^2}{\sqrt[4]{1-x^3}} dx$  (4)

c)  $\int \frac{\sin(\ln x)}{x} dx$ . (HINT: Use  $u = \ln x$ ) (2)



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**USE THIS PAGE TO ANSWER ANY QUESTION YOU MIGHT HAVE CANCELLED**

