UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

BACCALAUREUS OPTOMETRIAE

MODULE: **MATHEMATICS 1A**
MAT01A1

CAMPUS: **DFC**

ASSESSMENT: **JUNE EXAMINATION**

DATE: **31 MAY 2019** **TIME: 14:30 – 16:00**

ASSESSOR: **MR. IK LETLHAGE**

MODERATOR: **MR. EZ MORAPELI**

DURATION: 2 HOURS **MARKS: 70**

MARKS OBTAINED	_____ **70**	**%**

INITIALS AND SURNAME: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

THIS PAPER HAS 13 PAGES (INCLUDING THE COVER PAGE). ENSURE THAT YOU HAVE ALL THE PAGES.

INSTRUCTIONS: **ANSWER ALL QUESTIONS IN THE SPACES PROVIDED.**
USE ONLY A PEN FOR WRITING AND DRAWING (BLACK OR BLUE).
USE THE BLANK PAGES FOR ROUGH WORK AND MARK IT AS SUCH.
SHOULD YOU REQUIRE MORE SPACE FOR YOUR ANSWERS, USE THE BLANK PAGES. CLEARLY INDICATE THE RELEVANT QUESTION.
CALCULATORS ARE NOT ALLOWED.

QUESTION 1 [4]

Solve the given inequality and represent the solution in interval form.

$|4x+3| \geq 5$

QUESTION 2 [3]

Solve the equation for $x \in [0,\ 2\pi]$: $3\tan\frac{x}{2}+3=0.$

QUESTION 3 [3]

Find the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left[\left(\frac{i}{n}\right)^2-1\right]$

QUESTION 4 [4]

Find all the cube roots of $-64i,$ and the give the answers in the form $a+bi$

QUESTION 5 [2]

Suppose that it is known that $B \to \neg A$ is true, $\neg B \to \neg C$ is true and C is true. Determine the truth value of A. **Show how you reached your conclusion.**

QUESTION 6 [4]

Consider the statement: If n^2 is even, then n is even.

a) Write the contrapositive of this statement. (1)

b) Use proof by contraposition to prove the given statement. (3)

QUESTION 7 [2]

Let $f(x) = \log_3 x - 5$. Find the inverse of f: f^{-1}.

QUESTION 8 [4]

Find all the horizontal and vertical asymptotes of $f(x) = \dfrac{5x^2 + 3x - 8}{x^2 - 2x + 1}$

QUESTION 9 [3]

Consider the function $f(x)=\begin{cases} cx^2+2x \text{ if } x<2 \\ x^3-cx \text{ if } x\geq 2 \end{cases}$, c a constant.

Find the value of c that will make f continuous everywhere.

QUESTION 10 [3]

Use the Squeeze Theorem to find $\lim\limits_{x\to 0} x^2\cos\dfrac{1}{x}$.

QUESTION 11 [5]

Let $f(x)=\frac{1}{x^2}$.

a) Use the definition of the derivative of a function to find $f'(2)$. (3)

b) Find the equation of the tangent to the curve $y=\frac{1}{x^2}$ at the point $\left(2;\frac{1}{4}\right)$. (2)

QUESTION 12 [4]

Prove that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right),\ \forall x \in \mathbb{R}$.

QUESTION 13 [11]

Find the indicated derivative, in its simplest form

a) $\frac{dy}{dx}$ if $y = \frac{x^x \tan x}{e^{x^2}}$ (4)

b) $\frac{dy}{dx}$, using implicit differentiation, if $y^2 + x \ln y = 5x^3$ (3)

c) $\frac{d^2y}{dx^2}$ if $y = x\sec^{-1} x$ (4)

QUESTION 14 [3]

Use l'Hôspital's Rule to find $\lim_{x\to 0+} x\ln x$

QUESTION 15 [4]

Find f , if $f''(x) = x^3 + \sinh x,\ f'(0) = 1,\ f(2) = 2.6$.

QUESTION 16 [1]

Use the Fundamental Theorem of Calculus to find $\frac{d}{dx}\int_0^x e^{-t^2}\,dt$

QUESTION 17 [10]

Evaluate the following integrals. Show all the steps.

a) $\int_0^{\ln 3} e^{-x}\cosh 2x\,dx$ (4)

b) $\int \frac{5x^2}{\sqrt[4]{1-x^3}}dx$ (4)

c) $\int \frac{\sin(\ln x)}{x}dx$. (HINT: Use $u = \ln x$) (2)

USE THIS PAGE TO ANSWER ANY QUESTION YOU MIGHT HAVE CANCELLED