University of Johannesburg

Faculty of Science

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MAT0AA2

ENGINEERING MATHEMATICS 0AA2/2A2

EXAM

27 MAY 2019

Examiners: Dr. J. Mba
Dr E. Joubert

Internal Moderator: Dr. F. Schulz

Time: **120** minutes **60** marks

Surname and initials: ______________________________

Student number: ______________________________

Tel No.: ______________________________

INSTRUCTIONS:

1. The paper consists of **10** printed pages, **excluding** the front page.
2. Answer all questions.
3. **Write out all calculations (steps) and motivate all answers.**
4. Read the questions carefully.
5. Questions are to be answered on the question paper in the space provided. Please indicate when the blank side of a page is used.
6. **No calculators are allowed.**
7. Good luck!

Question 1 [10]

Answer the following **True and False** questions AND give a short justification (if True)/counterexample (if False):

a) If A is an $n \times n$ matrix and B is obtained from A by adding 2 times the first row of A to the third row, then $\det(B) = 2\det(A)$. (2)

TRUE	
FALSE	

b) For every $n \times n$ matrix A, $A \cdot \operatorname{adj}(A) = [\det(A)]I_n$, where I_n is the $n \times n$ identity matrix. (2)

TRUE	
FALSE	

c) Every subset of a vector space V that contains the zero vector in V is a subspace of V. (2)

TRUE	
FALSE	

d) The span of any finite set of vectors in a vector space is closed under addition and scalar multiplication. (2)

TRUE	
FALSE	

e) The set of upper triangular $n \times n$ matrices is a subspace of the vector space of all $n \times n$ matrices. (2)

TRUE	
FALSE	

Question 2 [4]

For which value(s) of a does the following system have zero solutions? One solution? Infinitely many solutions?

$$\begin{aligned} x_1 + x_2 + x_3 &= 4 \\ x_3 &= 2 \\ (a^2 - 4)x_3 &= a - 2 \end{aligned}$$

Question 3 [3]

a) Find a system of two linear equations in the variables x, y and z whose solutions are given parametrically by $x = 3 + t$, $y = t$ and $z = 7 - 2t$. (2)

b) Find another parametric solution to the same system in which the parameter is r, and $x = r$. (1)

Question 4 [3]

Let both A and B be $n \times n$ matrices. Prove that if A is invertible, then both $A + B$ and $I + BA^{-1}$ are invertible or both $A + B$ and $I + BA^{-1}$ are not invertible.

Question 5 [4]

Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 7 \\ 1 & 3 & 3 \\ -2 & -5 & 1 \end{bmatrix}.$$

Express A in the form $A = EFGR$, where E, F and G are elementary matrices, and R is a row-echelon form of A.

Question 6 [3]

Find all values of a, b, c and d for which A is skew-symmetric

$$A = \begin{bmatrix} 0 & 2a-3b+c & 3a-5b+5c \\ -2 & 0 & 5a-8b+6c \\ -3 & -5 & d \end{bmatrix}.$$

[Hint: $A^T = -A$ if A is skew-symmetric.]

Question 7 [3]

Prove that the equation of the line through the distinct points (a_1, b_1) and (a_2, b_2) can be written as det(A)=0, where

$$A = \begin{bmatrix} x & y & 1 \\ a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \end{bmatrix}.$$

Question 8 [3]

Consider the matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

. If it is given det(A)=-6, find the determinant of D, where

$$D = \begin{bmatrix} 3g & 3h & 3i \\ 2a+d & 2b+e & 2c+f \\ d & e & f \end{bmatrix}.$$

Question 9 [3]

Use Cramer's rule to solve for x' and y' in terms of x and y:

$$x = x' cos\theta - y' sin\theta$$
$$y = x' sin\theta + y' cos\theta$$

Question 10 [15]

Let $P(2, 3, -1)$, $Q = (-1, 2, -2)$ be two points and $\mathbf{n} = (1, -1, 2)$ be a vector in $\mathbb{R}^3$.

a) Find a point-normal equation of the plane $\mathcal{P}$ passing through the point P and having $\mathbf{n}$ as a normal. (2)

b) Verify that the point Q is not on the plane $\mathcal{P}$. (1)

c) Evaluate the distance between the point Q and the plane $\mathcal{P}$. (2)

d) Find parametric equations of the plane ($\mathcal{Q}$) containing the point Q and the vectors $\overrightarrow{QP}$ and $\mathbf{n}$. (3)

e) Find the area of the Parallelogram determined by the vectors $\overrightarrow{QP}$ and $\mathbf{n}$. (2)

f) Find a point $R(x, y, z)$ such that the vector $\overrightarrow{PR}$ is equivalent to the vector $\mathbf{n}$. (1)

g) Find $\text{Proj}_{\mathbf{n}}\overrightarrow{PQ}$. (2)

h) Show that the vectors $\overrightarrow{PQ}$, $\overrightarrow{PR}$ and $\overrightarrow{RQ}$ lie in the same plane. (2)

Question 11 [2]

Prove that the set $\{(a, b, c) \in \mathbb{R}^3 \mid a = -b + 2c\}$ is a subspace of $\mathbb{R}^3$.

Question 12 [7]

Let $B = \{(1, 0, 0), (2, 1, 0), (1, 1, -1)\}$ be a set of vectors in $\mathbb{R}^3$. Let $\mathbf{u} = (2, -1, 1)$.

a) Verify that B spans $\mathbb{R}^3$. (2)

b) Verify that B is a linearly independent set. (2)

c) Is B a basis for $\mathbb{R}^3$? Explain your answer. (1)

d) Find the coordinate vector of $\mathbf{u}$ relative to the basis B. (2)