**Surname: ____________________________**

**Initials: ______________________________**

**Student Number: ______________________**

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| **PROGRAM** | : | BEng<br>*MECHANICAL ENGINEERING SCIENCE* |
| **SUBJECT** | : | **INTRODUCTION TO ENGINEERING DESIGN 1B** |
| **CODE** | : | **IINEEB1/IIN1B21** |
| **DATE** | : | SUPPLEMENTARY EXAMINATION JANUARY 2020 |
| **DURATION** | : | 180 Minutes |
| **WEIGHT** | : | 50:50 |
| **TOTAL MARKS** | : | 80 |

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| **EXAMINER** | : | DR M BHAMJEE |
| **MODERATOR** | : | DR A MANESCHIJN |
| **NUMBER OF PAGES** | : | 25 PAGES (INCLUDING FORMULA SHEETS AND ROUGH WORK PAPER) |

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**INSTRUCTIONS**:

- DO NOT UNSTAPLE THESE SHEETS
- Answer all the questions
- Name and explain all assumptions where required
- Show all the steps in your calculations clearly where required

- One (1) mark per fact
- This is a Blackboard Test
- Ensure that you capture your answers on Blackboard and save regularly
- In addition, please copy the answers in the block provided on this question paper as a backup
- No answers will be graded from the hard copy
- The hard copy is a backup in the event that there is a Blackboard issue
- Rough Work Paper is provided at the end
- If you experience any errors with the Blackboard test please discuss with the head invigilators immediately.

**REQUIREMENTS**: ANSWER BOOKLETS
CALCULATOR

**QUESTION 1:** **[5]**

Explain the conditions for which each of the two theories, used to analyse friction clutches, is most applicable. (5)

**QUESTION 2:** **[5]**

Explain the concept of initial tension and how it is calculated. (5)

**QUESTION 3:** **[18]**

A flange coupling between two solid shafts of 50 mm diameter each with a length of 1 m transmits 1.2 kNm of torque. The Shear modulus for the shaft is 80 GPa. The flange coupling is held together with 6 bolts on a PCD of 300 mm. The flange coupling rotates at 150 $\text{min}^{-1}$. A rectangular key is used in the coupling. The shear stress in each bolt may not exceed 25 MPa and the shear stress in the key may not exceed 50 MPa. Maximum allowable compressive stress in the key is 100 MPa.

Calculate the following:

For the shaft:

$J =$ ________________________ $\text{m}^4$ (up to three decimal places) (2)
$\tau =$ ________________________ MPa (up to two decimal places) (2)
$\theta =$ ________________________ $^{\circ}$ (up to two decimal places) (2)

For the key:

$t =$ ________________________ mm (up to two decimal places) (2)
$w =$ ________________________ mm (up to two decimal places) (2)

Considering failure under shear:

$L =$ ____________________ mm (up to two decimal places) (2)

Considering failure under compression:

$L =$ ____________________ mm (up to two decimal places) (2)

For the bolts:

$\delta F =$ ________________________ N (up to two decimal places) (2)
$d =$ ________________________ mm (up to two decimal places) (2)

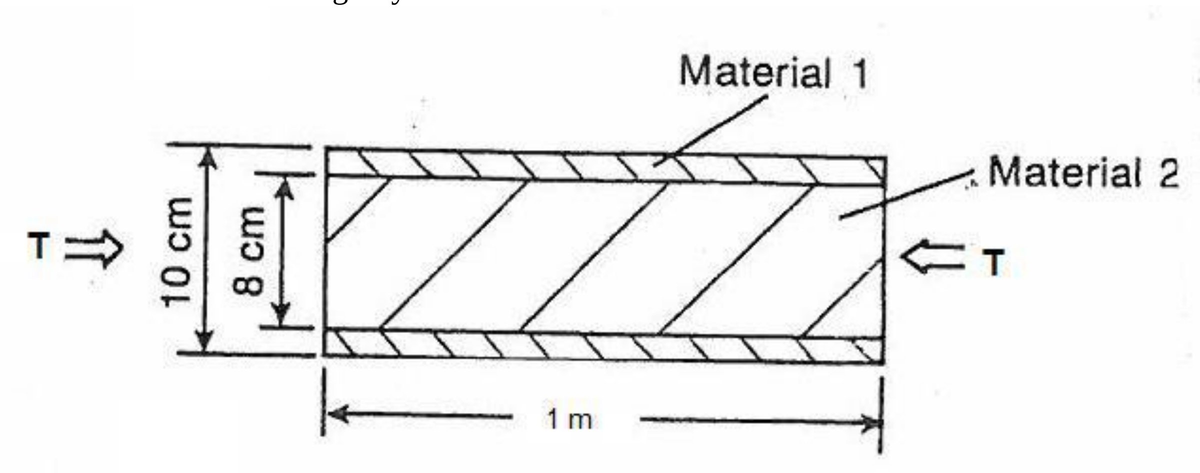
4/…

**QUESTION 4:** **[2]**

Based on your calculations from the previous question, which length value would you use for the key? Explain why you chose this value.

**QUESTION 5:** **[7]**

The compound shaft in Figure 1. is subjected to a torque of 2 kNm. Given the following values for the moduli of rigidity:




$G_1$ = 25 GPa and $G_2$ = 80 GPa

Figure 1. Compound shaft.

Determine the following values:

$\boldsymbol{J_1} =$ _______________ $\mathbf{10^{-6}}\ \boldsymbol{m^4}$ (up to three decimal places) (1)

$\boldsymbol{J_2} =$ _______________ $\mathbf{10^{-6}}\ \boldsymbol{m^4}$ (up to three decimal places) (1)

$\boldsymbol{T_1} =$ _______________ $\boldsymbol{Nm}$ (up to two decimal places) (1)

$\boldsymbol{T_2} =$ _______________ $\boldsymbol{Nm}$ (up to two decimal places) (1)

$\boldsymbol{\theta} =$ _______________ **º** (up to two decimal places) (1)

$\boldsymbol{\tau_1} =$ __________________ $\boldsymbol{MPa}$ (up to two decimal places) (1)

$\boldsymbol{\tau_2} =$ __________________ $\boldsymbol{MPa}$ (up to two decimal places) (1)

**QUESTION 6:** **[2]**

What is the equilibrium and the compatibility condition for the compound shafts in Question 5?

**QUESTION 7:** **[2]**

The width of a frictional material on a multiple disc clutch is (R-r) and is equal to $\frac{1}{3}$ of the maximum radius (R); $\mu = [u]$. Assume uniform pressure. The maximum diameter of the coupling is not to exceed [D] mm. The axial force is [W] kN. How many discs are required to transmit [P] kW at [N] min$^{-1}$?

**QUESTION 8:** **[5]**

Explain the method you used to get to the answer in the previous question. Use formulae to aid your explanation.

**QUESTION 9:** **[2]**

A leather-covered conical clutch transmits [P] kW at [N] rpm. The total conical angle is 20 deg. The width of the contact surface is [b] mm and the co-efficient of friction is 0.25. The average pressure is [r] kPa. Calculate the mean diameter in mm. (Provide the answer to two decimal places. Use . for the decimal separator and not , )

**QUESTION 10:** **[2]**

Explain the method you used to get to the answer in the previous question. Use formulae to aid your explanation.

**QUESTION 11:** **[2]**

Given a leather V-belt belt running at speed v $ms^{-1}$ with maximum tension $T_1$ in Newtons (N) on its tight side and minimum tension $T_2$ in Newtons (N) on its slack slide. What is the equation used to calculate the power P in Watts (W) which is being transmitted by the belt? (Use * for multiply, / for divide, + for addition and - for subtraction)

**QUESTION 12:** **[2]**

An open belt drive has pulleys with diameters [D] mm and [d] mm and a pulley centre distance of [c] m. Calculate the angle of wrap on the smaller diameter pulley in radians. (Provide the answer up to two decimal places).

**QUESTION 13:** **[2]**

What is the equation that you used to obtain the answer in Question 12?

**QUESTION 14:** **[2]**

An open belt drive with a V-belt is to be designed to transmit [P] kW at [N] rpm between two pulleys, having a pulley centre distance of [c] m. The driving pulley has a dimater of [d] mm and the angle of wrap is [a]$^o$. The allowable belt stress is [s] MPa, and belts are available having a thickness-to-width ratio of [r] and a material density of [p] kg/m3. Given that the co-efficient of friction between the belt and pulleys is [u] and the half-groove angle is [B]$^o$, calculate the minimum required belt width in mm. Provide the answer to two decimal places.

**QUESTION 15:** **[7]**

Explain the method you used to get to the answer in the previous question. Use formulae to aid your explanation.

**QUESTION 16:** **[15]**

Shown in Figure 2. is a three bar mechanism $O_1ABO_2$. For the shown position the link ($O_1A$) is 300 $min^{-1}$ clockwise. $O_1A$ = 60 mm, AB = 180 mm, $O_2B$ = 120 mm. **Use the following scale in your velocity diagram 1mm = 0.1885m/s**. Provide all answers up to two decimal places. Find:

$v_a =$ ______________________ m/s (5)
$v_b =$ ______________________ m/s (5)
$\omega_{ab} =$ ______________________ rad/s (5)

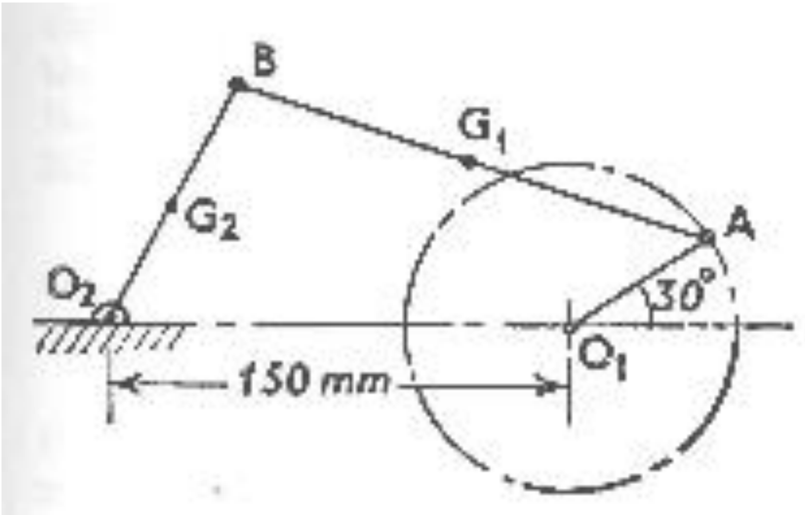



Figure 2. Three bar mechanism.

**Formula sheet**

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{2\tau}{D}$$

$$P = T \times \omega$$

$$\omega = \frac{2\pi N}{60}$$

$$t = \frac{D}{6}$$

$$w = \frac{D}{4}$$

$$J = \frac{\pi D^4}{32}$$

$$J = \frac{\pi(D^4 - d^4)}{32}$$

$$J = \frac{\pi R^4}{2}$$

$$J = \frac{\pi(R^4 - r^4)}{2}$$

$$\tau = \frac{Tr}{J}$$

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{30P}{\pi N}$$

$$T = n \times \delta F \times R$$

$$\delta F = \tau_b \times \frac{\pi d^2}{4}$$

$$\tau = \frac{F}{A_s}$$

$$= \frac{F}{\left(D/2\right) \times (WL)}$$

$$= \frac{2T}{DWL}$$

$$\tau_d = 0.5 \times \left(\frac{\sigma_y}{N}\right)$$

$$L = \frac{2T}{\tau_d DW}$$

$$\sigma_d = \frac{\sigma_y}{N}$$

$$\sigma = \frac{F}{A_c} = \frac{4T}{DLH}$$

$$L = \frac{4T}{\sigma_d DH}$$

$$T_1 = T_2$$

$$\theta_{Total} = \theta_1 + \theta_2$$

$$T_1 + T_2 = T$$

$$\theta_1 = \theta_2$$

$$W = \mu p \left(r_1^2 - r_2^2\right)$$

$$T = \frac{2}{3}\mu W \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right) \operatorname{cosec}\beta$$

$$W = 2\pi c \left(r_1 - r_2\right)$$

$$T = \pi \mu c\left(r_1^2 - r_2^2\right) \operatorname{cosec}\beta$$

$$= \frac{\mu W}{2}\left(r_1 + r_2\right) \operatorname{cosec}\beta$$

$$T = \mu W R \operatorname{cosec}\beta$$

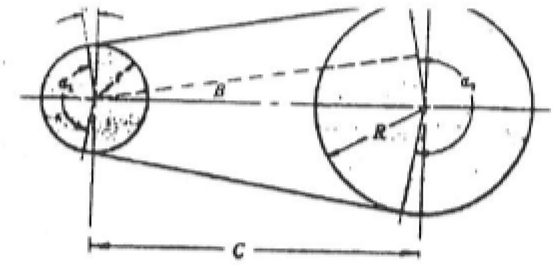
$$W = p \times \pi\left(r_1^2 - r_2^2\right)$$
$$T = \frac{2}{3}\pi\mu p\left(r_1^3 - r_2^3\right)$$
$$T = \frac{2}{3}\mu W\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)$$

$$W = 2\pi c\left(r_1 - r_2\right)$$
$$T = \pi\mu c\left(r_1^2 - r_2^2\right)$$
$$T = \mu W\frac{\left(r_1 + r_2\right)}{2} = \mu W R$$

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{(Eitelwein's equation)} \quad (14.3)$$

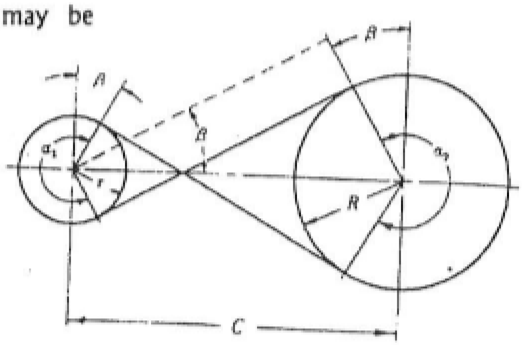
$$\text{Power} = \left(T_1 - T_2\right)v, [W] \qquad (14.1)$$

$$\sin\beta = \frac{R - r}{C}$$
$$\alpha_1 = 180° - 2\beta = 180° - 2\sin^{-1}\frac{R - r}{C},$$
$$\alpha_2 = 180° + 2\beta = 180° + 2\sin^{-1}\frac{R - r}{C}$$




The angles of wrap for a crossed belt drive may be determined by:

$$\sin\beta = \frac{R + r}{C}$$
$$\alpha_1 = \alpha_2 = 180° + 2\beta$$
$$= 180° + 2\sin^{-1}\frac{R + r}{C}$$



then power transmitted $= (T_1 - T_2)\nu\ W$ (14.4)

$$= T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)\nu\ W \qquad (14.5)$$

$$N = \frac{R}{2}\operatorname{cosec}\beta$$

$\therefore$ frictional resistance $= 2\ \mu N$

$$= \mu R \operatorname{cosec}\beta$$

$$\frac{T_1}{T_2} = e^{\mu\theta \operatorname{cosec}\beta} \qquad (14.6)$$

$$T_C = m\nu^2 \qquad (14.7)$$

$$\frac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta} \text{ or } e^{\mu\theta \operatorname{cosec}\beta} \qquad (14.8)$$

$$\text{power} = (T_1 - T_c)\left(1 - \frac{1}{e^{\mu\theta}}\right)\nu \qquad (14.9)$$

$$m\nu^2 = \frac{1}{3}T_1$$

$$T_c = \frac{1}{3}T_1 \qquad (14.10)$$

$$T_1 - T_2 = 2T_o \qquad (14.11)$$

$$\frac{\sigma_1 \boxed{-} m'\nu^2}{\sigma_2 - m'\nu^2} = e^{\mu\theta}$$

$$\frac{T_1 - T_2}{\sigma_1 - \sigma_2} = \text{required cross section area}$$

$$\frac{T_1 - mv^2}{T_2 - mv^2} = e^{\mu\theta/\sin\frac{1}{2}\theta}, \text{ where } m = bt\rho$$

$$L = \frac{\pi}{2}(2r_1 + 2r_2) + 2AB + \frac{1}{4AB}(2r_1 + 2r_2)^2$$

$$L = r_1(2\pi - 2\varphi) + r_2(2\pi - 2\varphi) + 2r_1 tan\varphi + 2\mathrm{r}_2\tan\varphi$$

$$L = \frac{\pi}{2}(2r_1 + 2r_2) + 2AB + \frac{1}{4AB}(2r_1 - 2r_2)^2$$

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} \quad \text{or} \quad \alpha = \frac{d^2\theta}{dt} = \ddot{\theta}$$

$$\omega\, d\omega = \alpha\, d\theta \quad \text{or} \quad \dot{\theta}\, d\dot{\theta} = \ddot{\theta}\, d\theta$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\boxed{\begin{aligned} v &= r\omega \\ a_n &= r\omega^2 = v^2 / r = v\omega \\ a_t &= r\alpha \end{aligned}}$$

$$\left[v^2 = 2as\right]$$

$$\left[a_n = v^2 / r\right]$$

$$\left[a = \sqrt{a_n^2 + a_t^2}\right]$$

$$\omega = \frac{d\theta}{dt} \quad [\text{rad/s}]$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$\boxed{v_{A/B} = r\omega}$$

$$\therefore \frac{v_{CA}}{v_{BA}} = \frac{\omega AC}{\omega AB}$$

$$\therefore \boxed{\frac{ac}{ab} = \frac{AC}{AB}}$$

$$\omega = \frac{v_{AB}}{AB}$$

**ROUGH WORK PAPER:**