PROGRAM	:	BACCALAUREUS TECHNOLOGIAE *CHEMICAL ENGINEERING*
SUBJECT	:	**CONTROL SYSTEMS IV**
CODE	:	**ICP4111**
DATE	:	SUPPLEMENTARY EXAMINATION 2019
DURATION	:	3 HOURS (8:30 – 11:30)
WEIGHT	:	40 : 60
TOTAL MARKS	:	80

EXAMINER(S)	:	DR A.N. MATHERI & DR T. MASHIFANA
MODERATOR	:	DR T. J. PILUSA
NUMBER OF PAGES	:	07 PAGES

INSTRUCTIONS	:	QUESTION PAPERS MUST BE HANDED IN.
REQUIREMENTS	:	Use of scientific (non-programmable) calculator is permitted (only one per candidate): Graph Paper

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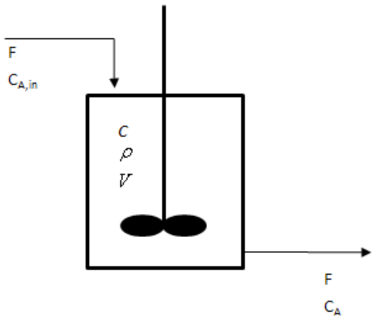
INSTRUCTIONS TO CANDIDATES:

- Purpose of assessment is to determine not only if you can write down an answer, but also to assess whether you understand the concept, principles and expressions involved. Set out solution in a logical and concise manner with justification for the steps followed.
- **ATTEMPT ALL QUESTIONS.** Please answer each question to the best of your ability.
- Write your details (module name and code, ID number, student number etc.) on script (s).
- Number each question clearly; questions may be answered in any order.
- Make sure that you read each question carefully before attempting to answer the question.
- Show all steps (and units) in calculations; this is a 'closed book' test.
- Ensure your responses are legible, clear and include relevant units (where appropriate).
- **ANNEXURE 1:** Formulas and Laplace Table

Question 1 **[20 marks]**

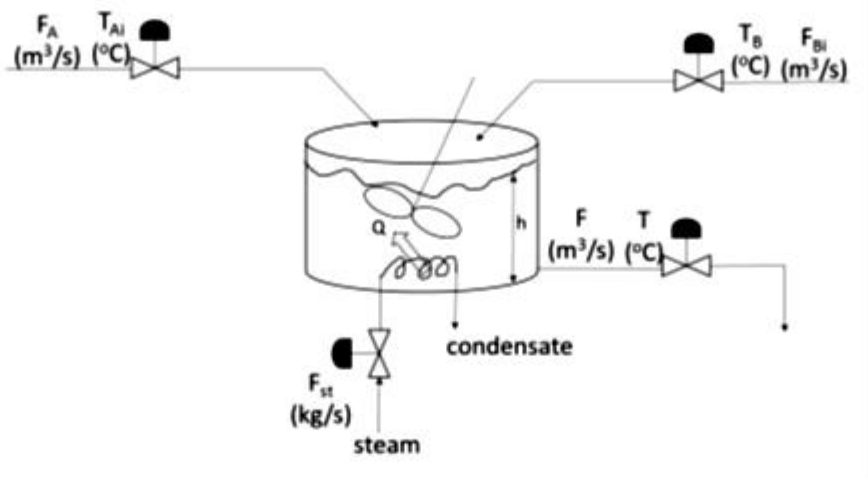
The following ***second*** order reaction A→B+C takes place in an isothermal CSTR of volume V (See drawing below). The reaction rate constant is denoted **k** and the concentration of A (**$C_{A,in}$**) in the inlet stream changes from time to time. Assume that the volume **V** of the CSTR, the medium density **ρ** and the flow **F** rate into and out of the CSTR remain constant.

1. Using the component, A balance, derive the mathematical model of this system**;** **[2 Marks]**
2. Is the model linear or non-linear? If the resulting model is non-linear, approximate it to a linear model around the steady state concentration of A ($C_{A,S}$) and determine the system time constant in terms of reactor volume, liquid flowrate, reaction rate constant and steady state concentration of component A; **[6 Marks]**
3. Derive the transfer function relating the deviation of the inlet concentration of A (C_{Ai}) to the deviation of the outlet concentration of A (C_A). **[12 Marks]**



Question 2 **[20 marks]**

Consider the tank heater system shown in below. Two liquid streams A and B enter the tank at flowrates FAi and FBi all in m^3/s and at temperatures TAi and TBi all in ^oC, respectively. The liquid streams are mixed and heated with steam (having a flowrate of Fst, kg/s). Let F (m^3/s) and T (^oC) be the flowrate and temperature of the steam leaving the tank. The tank is considered to be well stirred, which means that the temperature of the effluent is equal to the temperature of the liquid in the tank. The operational objective of this heater is to keep the effluent temperature T at the desired value Ts.

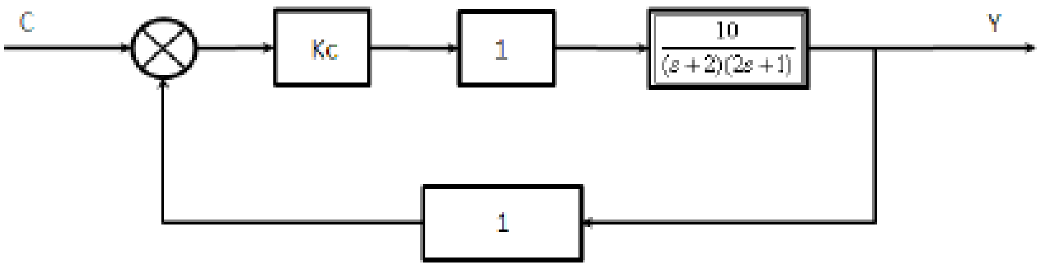


Devise the theoretical model for this process, identifying all the required parameters as per the four steps of modelling principles.

Question 3 **[20 marks]**

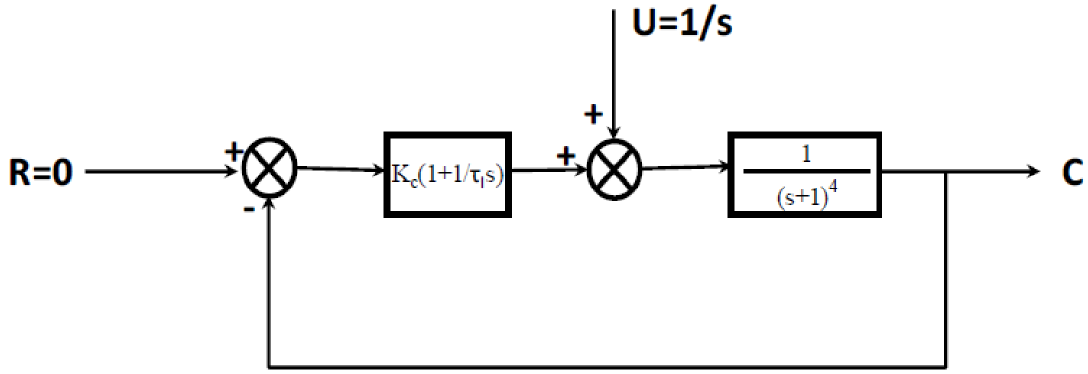
Select the gain of a proportional controller in the closed loop shown below using one quarter decay ratio criterion. The process is described by:

$$G_p(s) = \frac{10}{(s+2)(2s+1)}$$



Question 4 **[20 marks]**

For the control system shown below, determine the controller settings for a PI controller using the Z-N method.



END

ANNEXURE 1

$$Overshoot = exp(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}})$$

$$Decay\ ratio = exp(\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}})$$

$$T = \frac{2\pi\tau}{\sqrt{1-\zeta^2}}$$

$$u = K_c\varepsilon + b$$

$$u = K_c\varepsilon + \frac{K_c}{\tau_I}\int_0^t \varepsilon dt + b$$

$$u = K_c\varepsilon + K_c\tau_D\frac{d\varepsilon}{dt} + b$$

$$u = K_c\varepsilon + K_c\tau_D\frac{d\varepsilon}{dt} + \frac{K_C}{\tau_I}\int_0^t \varepsilon dt + b$$

$$G_{PRC}(s) = \frac{y'_m(s)}{c'_m} \approx \frac{Ke^{-t_d s}}{\tau s + 1}$$

1. For proportional controllers:

$$K_c = \frac{1}{K}\frac{\tau}{t_d}(1+\frac{t_d}{3\tau})$$

2. For PI controllers:

$$K_c = \frac{1}{K}\frac{\tau}{t_d}(0.9+\frac{t_d}{12\tau})$$

$$\tau_I = t_d \frac{30+3\frac{t_d}{\tau}}{9+20\frac{t_d}{\tau}}$$

3. For PID controllers

$$K_c = \frac{1}{K}\frac{\tau}{t_d}(\frac{4}{3}+\frac{t_d}{4\tau})$$

$$\tau_I = t_d \frac{32+6\frac{t_d}{\tau}}{13+8\frac{t_d}{\tau}}$$

$$\tau_D = t_d \frac{4}{11+2\frac{t_d}{\tau}}$$

$$Amplitude\ ratio\ AR = Modulus\ G(j\omega) = \frac{K_p}{\sqrt{1+\omega^2\tau_p{}^2}}$$

$$Phase\ lag\ \varphi = \tan^{-1}\left(-\omega\tau_p\right)$$

$$The\ modulus\ for\ G_p(j\omega)\ is\ \left|G_p(j\omega)\right| = |Z_1||Z_2||Z_3|$$

$$The\ phase\ lag\ is\ \varphi = \varphi_1 + \varphi_2 + \varphi_3$$

$$\text{Ultimate gain} = K_u = \frac{1}{M}$$

$$\text{Ultimate period of sustained cycling} = P_u = \frac{2\pi}{\omega_{co}} \quad \text{min/ cycle}$$

	K_c	τ_I min)	τ_D (min)
Proportional (P)	$K_u/2$	-	-
Proportional-integral (PI)	$K_u/2.2$	$P_u/1.2$	-
Proportionall-integral-derivative (PID)	$K_u/1.7$	$P_u/2$	$P_u/8$

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\ n=1,2,3,\ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\ n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$	e^{-cs}	26.	$\delta(t-c)$	e^{-cs}