



PROGRAM : BACCALAUREUS TECHNOLOGIAE
CHEMICAL ENGINEERING

SUBJECT : CONTROL SYSTEMS IV

CODE : ICP4111

DATE : SUMMER MAIN EXAMINATION 2019

DURATION : 3 HOURS (8:30 – 11:30)

WEIGHT : 40 : 60

TOTAL MARKS : 80

EXAMINER(S) : DR A.N. MATHERI & DR T. MASHIFANA

MODERATOR : DR T.J. PILUSA

NUMBER OF PAGES : 07 PAGES

INSTRUCTIONS : QUESTION PAPERS MUST BE HANDED IN.

REQUIREMENTS : Use of scientific (non-programmable) calculator is permitted
(only one per candidate); Graph Paper

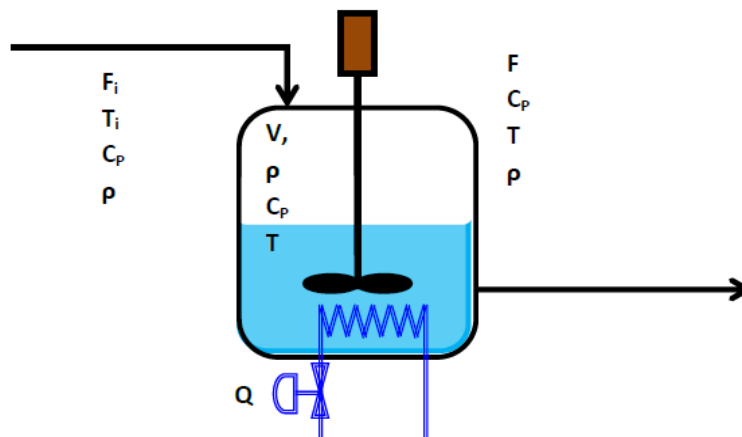
INSTRUCTIONS TO CANDIDATES:

- Purpose of assessment is to determine not only if you can write down an answer, but also to assess whether you understand the concept, principles and expressions involved. Set out solution in a logical and concise manner with justification for the steps followed.
- **ATTEMPT ALL QUESTIONS.** Please answer each question to the best of your ability.
- Write your details (module name and code, ID number, student number etc.) on script (s).
- Number each question clearly; questions may be answered in any order.
- Make sure that you read each question carefully before attempting to answer the question.
- Show all steps (and units) in calculations; this is a 'closed book' test.
- Ensure your responses are legible, clear and include relevant units (where appropriate).
- **ANNEXURE 1:** Formulas and Laplace Table

Question 1

[25 marks]

Consider a stirred-tank heater represented by Figure below. A liquid stream, with conditions dictated by up-stream processes, enters an insulated well-stirred tank at a flow rate F_i (mass/time) and temperature T_i . It is desired to maintain (or control) the temperature in the tank at T_s using a programmable logic controller. If the measured tank temperature T_m differs from the desired set-point temperature T_s , the controller senses the deviation $T_e = T_s - T_m$, and changes the heat input Q (energy/time), in such a way as to reduce the magnitude of the deviation.



The heat, Q is changed by manipulating the flow of saturated steam, F_s (mass/time). The heat added to the liquid process stream can be calculated from the heat of condensation of steam as

$$Q = \lambda_s F_s.$$

1.1 If the flow of the process liquid, F_i is constant and $F = F_i$, answer the following question:

- Derive the time domain process dynamics differential model for the system in its standard form in deviation variables for Temperature, T . [7 Marks]
- Give the Laplace domain expression of the system i.e. Transfer function of the system. [6 Marks]
- Draw a block diagram of the system showing the input(s), transfer block(s) and outputs(s). [2 Marks]

1.2 Consider a stirred-tank heater represented by Figure above of the process derived with the following process parameters:

$$F_i = 100 \text{ kg/hr}, T_i = 20^\circ \text{C}, C_p = 4.2 \text{ kJ/(kg}\cdot\text{K)}, \lambda_s = 2000 \text{ kJ/kg}, V = 5000 \text{ m}^3, \rho = 1000 \text{ kg/m}^3$$

If the process is at steady state with $T = T_s = 50^\circ \text{C}$, answer the following questions:

- Calculate the flow of steam, F_s required to supply the heat in order to maintain T at the given set-point value, T_s . [8 Marks]
- Calculate the heat flow required (kJ/hr) to maintain T at the given value. [2 Marks]

Question 2**[15 marks]**

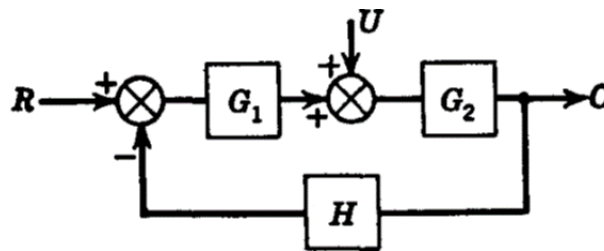
A pneumatic PI controller has an output pressure of 10 psi when the set point and pen point are together. The set point and pen point are suddenly displaced by 0.5 in. (i.e., a step change in error is introduced) and the following data are obtained:

Time, Sec	Psig
0	10
0 ⁺	8
20	7
60	5
90	3.5

- Determine the actual gain (psig per inch displacement) and the integral time. [6 marks]
- A unit-step change in error is introduced into a PID controller. If $K_C = 10$, $\tau_I = 1$, and $\tau_D = 0.5$, plot the response of the controller, $P(t)$. [9 Marks].

Question 3**[20 marks]**

Study the stability of the system below using the root-locus analysis. Draw the root-locus diagram using graph for the following values of K_c ranging from 0 to 20. In particular pay attention to these values of K_c 0; 0.04; 0.07; 0.26; 1.10; 4.42; 10.00 and 16.67. Determine the value of K_c that will keep the system stable.



$$G_1 = K_c$$

$$G_2 = \frac{1}{(s+1)\left(\frac{s}{2}+1\right)}$$

$$H = \frac{1}{\frac{s}{3}+1}$$

Question 4**[20 marks]**

The transfer functions of the process, measuring device and final control element of a control system are respectively given as:

$$G_p = \frac{1}{(5s+1)(2s+1)}$$

$$G_m = \frac{1}{10s+1}$$

$$G_f = 1.0$$

Determine the controller settings using the Ziegler-Nichols tuning method for P, PI and PID controller.

END

ANNEXURE 1

$$\text{Overshoot} = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

$$\text{Decay ratio} = \exp\left(\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

$$T = \frac{2\pi\tau}{\sqrt{1-\zeta^2}}$$

$$u = K_c \varepsilon + b$$

$$u = K_c \varepsilon + \frac{K_c}{\tau_I} \int_0^t \varepsilon dt + b$$

$$u = K_c \varepsilon + K_c \tau_D \frac{d\varepsilon}{dt} + b$$

$$u = K_c \varepsilon + K_c \tau_D \frac{d\varepsilon}{dt} + \frac{K_c}{\tau_I} \int_0^t \varepsilon dt + b$$

$$G_{PRC}(s) = \frac{y'_m(s)}{c'_m} \approx \frac{K e^{-t_d s}}{\tau s + 1}$$

1. For proportional controllers:

$$K_c = \frac{1}{K} \frac{\tau}{t_d} \left(1 + \frac{t_d}{3\tau}\right)$$

2. For PI controllers:

$$K_c = \frac{1}{K} \frac{\tau}{t_d} \left(0.9 + \frac{t_d}{12\tau}\right)$$

$$\tau_I = t_d \frac{30 + 3 \frac{t_d}{\tau}}{9 + 20 \frac{t_d}{\tau}}$$

3. For PID controllers

$$K_c = \frac{1}{K} \frac{\tau}{t_d} \left(\frac{4}{3} + \frac{t_d}{4\tau}\right)$$

$$\tau_I = t_d \frac{32 + 6 \frac{t_d}{\tau}}{13 + 8 \frac{t_d}{\tau}}$$

$$\tau_D = t_d \frac{4}{11 + 2 \frac{t_d}{\tau}}$$

$$\text{Amplitude ratio } AR = \text{Modulus } G(j\omega) = \frac{K_p}{\sqrt{1 + \omega^2 \tau_p^2}}$$

$$\text{Phase lag } \varphi = \tan^{-1}(-\omega \tau_p)$$

$$\text{The modulus for } G_p(j\omega) \text{ is } |G_p(j\omega)| = |Z_1| |Z_2| |Z_3|$$

$$\text{The phase lag is } \varphi = \varphi_1 + \varphi_2 + \varphi_3$$

$$\text{Ultimate gain} = K_u = \frac{1}{M}$$

$$\text{Ultimate period of sustained cycling} = P_u = \frac{2\pi}{\omega_{co}} \quad \text{min/cycle}$$

	K_c	τ_I (min)	τ_D (min)
Proportional (P)	$K_u/2$	-	-
Proportional-integral (PI)	$K_u/2.2$	$P_u/1.2$	-
Proportionall-integral-derivative (PID)	$K_u/1.7$	$P_u/2$	$P_u/8$

Table of Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$	2. e^{at}	$\frac{1}{s-a}$
3. $t^n, n=1,2,3,\dots$	$\frac{n!}{s^{n+1}}$	4. $t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. \sqrt{t}	$\frac{\sqrt{\pi}}{2s^{3/2}}$	6. $t^{n-1/2}, n=1,2,3,\dots$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+1/2}}$
7. $\sin(at)$	$\frac{a}{s^2+a^2}$	8. $\cos(at)$	$\frac{s}{s^2+a^2}$
9. $t \sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10. $t \cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11. $\sin(at) - at \cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12. $\sin(at) + at \cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13. $\cos(at) - at \sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14. $\cos(at) + at \sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15. $\sin(at+b)$	$\frac{s \sin(b) + a \cos(b)}{s^2+a^2}$	16. $\cos(at+b)$	$\frac{s \cos(b) - a \sin(b)}{s^2+a^2}$
17. $\sinh(at)$	$\frac{a}{s^2-a^2}$	18. $\cosh(at)$	$\frac{s}{s^2-a^2}$
19. $e^{at} \sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20. $e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21. $e^{at} \sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22. $e^{at} \cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23. $t^n e^{at}, n=1,2,3,\dots$	$\frac{n!}{(s-a)^{n+1}}$	24. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
25. $u_c(t) = u(t-c)$	$\frac{e^{-cs}}{s}$	26. $\delta(t-c)$	e^{-cs}