FACULTY OF SCIENCE

Department of Pure and Applied Mathematics **Module MAT1C** **Bio and Enviro Maths and Stats** **Campus: APK** **Assessment: Supplementary Summative Assessment**

Date: Time:
Assessor   Andrew Einhorn
Internal Moderator   S Mafunda
Duration 120 minutes   Marks 100
Number of pages: 1-12 pages

FIRST + LAST NAME   _____

STUDENT NUMBER   _____

CONTACT NUMBER   _____

TUTORIAL GROUP   _____

INSTRUCTIONS

1. **Answer all of the questions on the paper in pen. You may use pencils to sketch graphs or draw diagrams.**
2. **Calculators may be used on this test. Both a poisson and normal table of probabilities are provided in this paper.**
3. **Show all calculations and motivate all answers.**
4. **If you require extra space, continue your work on the <u>adjacent</u> blank page and indicate this clearly.**
5. **Unless stated otherwise, round all decimal answers to 2 significant figures.**

Question 1 [5 marks]

For questions 1.1 through 1.5, mark the correct answers with an X in the box below.

	A	B	C	D	E
1.1					
1.2					
1.3					
1.4					
1.5					

1.1. An angle of 135° has a radian measure of:

a. $\frac{3\pi}{2}$ b. $\frac{3\pi}{4}$ c. $\frac{3\pi}{7}$ d. $\frac{3\pi}{5}$ e. None of these

1.2. Which of the following functions is not increasing on its domain

   a. $f(x) = 2^x$ b. $f(x) = \tan(x)$ c. $f(x) = \log_{10} x$

   d. $f(x) = x^2$ e. $f(x) = x^3$

1.3. The coefficient of the b^2 in the expansion of $(a + 2b)^3$ is:

   a. 2   b. 4   c. 6

   d. 8   e. 12

1.4. Which of the following is true:

   a. If f is a function, then $f(s + t) = f(s) + f(t)$
   b. If $f(s) = f(t)$ then $s = t$
   c. If $x_1 < x_2$ and f is a decreasing function, then $f(x_1) > f(x_2)$
   d. If f is a function, then $f(3x) = 3f(x)$
   e. If f and g are functions, then $f \circ g = g \circ f$

1.5. Which of the following functions reflect $ln(x)$ about the y-axis and stretch it horizontally by a factor of 2?

   a. $y = \ln\left(-\frac{x}{2}\right)$ b. $y = -\ln(2x)$ c. $y = \frac{1}{2}\ln(-x)$

   d. $y = -\ln\left(\frac{x}{2}\right)$ e. $y = -2\ln(x)$

<u>Question 2 [10 marks]</u>

2.1 Find the domain of the following functions:

   a. $f(x) = \log_2(2x - 3)$

(1)

   b. $g(x) = \frac{1}{\sqrt{x^2 - 9}}$

(2)

2.2 Use the table to evaluate each expression:

x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

   a. $f(g(1))$

   b. $g(f(1))$

   c. $(g \circ f)(3)$

   d. $(f \circ g)(6)$

(4)

2.3 Given $f(x) = \ln(\cos x)$, find $f''\left(\frac{\pi}{6}\right)$

(3)

<u>Question 3 [10 marks]</u>

3.1 Given the function $f(x) = x^3 - 5x$

    a. Find the $f'(x)$ using the definition of the derivative (i.e. from first principles)

(4)

    b. Find the equation of the tangent line to f when $x = 1$.

(3)

    c. Find the equation of the normal line when $x = 1$.

(1)

3.2 If an equation of the tangent line to the graph of $y = f(x)$ at the point where $x = 2$ is $y = 4x - 5$, find $f(2)$ and $f'(2)$.

(2)

3/12

Question 4 [10 marks]

Sketch the following functions, making sure to indicate any x- or y- intercepts, the scale of your axes, and any applicable asymptotes.

a. $y = \sin(2x) + 1$ on the interval $[0, 2\pi]$

(2.5)

b. $f(x) = -(x + 1)^3$

(2.5)

c. $y = -\log_2(x + 2)$

(2.5)

d. $f(x) = 2(3^{-x})$

(2.5)

<u>Question 5 [15 marks]</u>

5.1 Find the derivatives of the following functions:

    a. $y = x^2 \sin(\pi x)$

      (2)

    b. $y = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}}$

      (2)

    c. $f(\theta) = \cos(tan(\theta))$

      (2)

    d. $f(x) = 3^{xln(x)}$

      (2)

e. $y = \dfrac{e^{\frac{1}{x}}}{x^2}$

(2)

f. $y = \sqrt{\sin(\sqrt{x})}$

(2)

5.2 Prove that $\dfrac{d}{dx}\csc(x) = -\csc(x)\cot(x)$ by rewriting $\csc(x)$ in terms of basic trig function(s) and applying the quotient rule to find its derivative.

(3)

<u>Question 6 [5 marks]</u>

For questions 6.1 through 6.5, mark the correct answers with an X in the box below.

	A	B	C	D	E
1					
2					
3					
4					
5					

6.1   The pages of a book are labeled 1 up to 25. If I flip to a random page, what is the probability that the page number will contain a 2?

     a. $\frac{1}{25}$ b. $\frac{3}{25}$ c. $\frac{5}{25}$ d. $\frac{8}{25}$ e. $\frac{9}{25}$

6.2.   Thandi has 7 different pairs of shoes in her closet, but can only fit 4 of them into her bag for her holiday. How many different combinations of pairs of shoe can she pack in her bag?

     a. 4   b. 28   c. 35

     d. 210   e. 840

6.3.   If there are 3 Red balls and 4 Green balls in a bag, and three balls are drawn consecutively without replacement, what is the probability that all three will be red?

     a. $\frac{1}{35}$ b. $\frac{1}{8}$ c. $\frac{3}{7}$

     d. $\frac{1}{7}$ e. $\frac{81}{243}$

6.4.   If the probability of A and B occurring is $\frac{1}{6}$, the probability of A or B occuring is $\frac{2}{3}$, and $P(B) = \frac{1}{4}$, then $P(A)$ is equal to:

     a. $\frac{1}{2}$ b. $\frac{7}{12}$ c. $\frac{1}{6}$

     d. $\frac{3}{4}$ e. $\frac{3}{8}$

6.5. If $p(A \cap B) = 0$, then which of the following statements are true:

           i.   A and B are independent
           ii.   A and B are mutually exclusive
           iii. $p(A \cup B) = p(A) + p(B)$

     a.   (i) only
     b.   (i) and (ii)
     c.   (ii) and (iii)
     d.   (i) and (iii)
     e.   (i), (ii) and (iii)

Question 7 [10 marks]

7.1. The peak temperatures measured in Cape Town for the 30 days in December are tabulated below (℃) in ascending order.

21.3	22.2	23.5	24.1	24.7	25.1
25.6	26.2	26.7	26.9	27.4	27.5
28.2	28.7	29.6	29.9	30.5	30.7
31.2	31.5	31.5	31.9	32.8	33.0
34.5	34.6	35.7	36.2	36.7	38.3

a. Use this data to complete the table below. Begin your first interval at 20, use a class width of 3, and use the rule of left inclusion for your intervals.

Interval	Frequency	Relative Freq (2 d.p)

(3)

b. Use this data to construct a relative frequency histogram below. Label your axes and give your chart an appropriate title.

(3)

c. If this data is representative of a normal December, what percentage of days have a peak temperature of more than 29 degrees?

(1)

7.2 The lowest recorded temperature recorded in South Africa is tabulated for the five years from 2013 to 2017. Compute the mean and std dev for this data (show all working).

2013	2014	2015	2016	2017
−20℃	−15℃	−12℃	−16℃	−17℃

(3)

<u>Question 8 [10 marks]</u>

8.1. In a survey of university students, a marketing professional asks students if they prefer Marvel Comics or DC Comics. She finds the ratio of preferences is 6:4 (Marvel to DC). If 3 students are selected at random, and we define X to be the number of Marvel fans.

   a.   Compute the following probabilities to 2 d.p.

   $P(X = 0) =$

   $P(X = 1) =$

   $P(X = 2) =$

   $P(X = 3) =$ (2)

   b.   In the space below, construct the probability distribution (i.e. draw a bar chart of these probabilities). Label both axes and give your chart a title.

                                                                                                    (2)

   c.   What is the probability that at least one student will be a Marvel fan?

                                                                                                    (1)

8.2. A social scientist is researching student interest in politics. She finds that 40% of students voted in the national election. Suppose she takes a random sample of 18 students and asks them if they voted in the national election.

   a.   Given that $X \sim B(18, 0.4)$, find $E(X)$ and $\sigma(X)$.

                                                                                                    (2)

   b.   Use the normal approximation to compute the probability that fewer that 10 of them voted in the election.

                                                                                                    (3)

<u>Question 9 [10 marks]</u>

9.1 A university computer lab has an average of 7 visitors every hour. Let X be the number of visitors to the lab in any given hour, and suppose $X \sim P_0(7)$

a. Find the probability that there will be exactly 8 visitors in a given hour.

(2)

b. Find the probability that there will be more than 15 visitors in a two hour period.

(3)

c. What is the probability of there being fewer than 3 visitors in a 30 minute period?

(2)

9.2 In a factory manufacturing light bulbs there is a 1.5% probability of a light bulb being defective. Use an appropriate approximation to compute the probability that in a batch of 300 lightbulbs, more than 5 of them will be defective (to 3 d.p).

(3)

Question 10 [15 marks]

10.1 The Fire Creek mine in Nevada, United States, produces the highest grade of gold ore in the world. The amount of gold per ton of ore is normally distributed, with a mean of $\mu = 44g/ton$ and a standard deviation $\sigma = 9.5\ g/ton$.

a. What is the probability that a given ton of gold ore will contain more than 55g of gold?

(2)

b. What proportion of the ore from Fire Creek will have a grade between 30-40g/ton?

(3)

c. If the top (i.e. richest) 15% of ore samples are processed twice in order to extract all of the gold, what grade limit should be set in order to select such samples for double-processing? [Hint: Draw a picture]

(2)

d. Within what size range of μ will 90% of samples fall? (i.e. Find a such that the range $\mu - a$ to $\mu + a$ contains 90% of the distribution).

(3)

10.2   The mine management find that on average there is a 5% probability of an accident of occurring on any given day. Use the normal approximation to compute the probability that in a 360 day period, there will be no more than 15 accidents.

(5)