UNIVERSITY
OF
JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE   MAT100
                BUSINESS MATHEMATICS

CAMPUS   APK
ASSESSMENT   EXAM

DATE 25/05/2019   TIME 08:30

ASSESSOR(S)   MR T. MUDZIIRI SHUMBA

MODERATOR   DR A. SWARTZ

DURATION 120 MINUTES   MARKS 75

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 10 PAGES

INSTRUCTIONS:

1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.

2. CALCULATORS ARE ALLOWED.

3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.

4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE <u>FACING</u> BLANK PAGE AND INDICATE THIS CLEARLY.

<u>Question 1</u> [14 marks]

1.1 Simplify the following expression completely:

$$\frac{3x+3}{x^2+3x+2} \div \frac{x^2-x}{x^2+x-2}$$

$\boxed{2}$

1.2 Solve the equation $x^{-4} - 9x^{-2} + 20 = 0$ and express all answers in a form such that any denominators are free of radicals. $\boxed{4}$

1.3 Solve the inequality $|x - \frac{1}{2}| > \frac{1}{2}$ giving your answer in interval notation. $\boxed{3}$

1.4 State the domain of the function $f(x) = \sqrt{x - 1}$. ☐1

1.5 Solve the following logarithmic equation: $\log_3(x + 2) + \log_9(x + 2)^2 = 2$. ☐4

Question 2 [11 marks]

2.1 Let $B = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -4 & 0 \end{bmatrix}, E = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}, F = \begin{bmatrix} \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$. Find

(a) $6F + EI_3$. ☐3

(b) B^T. ☐1

2.2 (a) Solve the system of equations below using the row reduction method. If the system has an infinite number of solutions, give the general solution in parametric form:

$$\begin{cases} x & + & y & + & z & = & 0 \\ x & & & - & z & = & 0 \\ x & - & 2y & - & 5z & = & 0 \end{cases}$$

5

(b) Using your answer to (a), or otherwise, determine whether the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & -5 \end{bmatrix}$

is invertible (i.e has an inverse).

2

Question 3 [7 marks]

Maximize $P = 2x + 5y$ subject to

$$
\begin{array}{rcrcl}
2x & + & y & \leq & 16 \\
x & + & y & \leq & 4 \\
  &   & x & \leq & 1
\end{array}
$$

$$x \geq 0, y \geq 0$$

using the method of corners. Shade the **unwanted** region.   7

Question 4 [11 marks]

4.1 The present value A, of an annuity of payments R per period, at a periodic rate of interest r over n periods is given by

$$A = R \cdot \frac{1 - (1 + r)^{-n}}{r}.$$

Express the number of periods (n) in terms of the other variables. $\boxed{3}$

4.2 The commerce society would like to endow a prize of R1200 to the student who is deemed to have exhibited the most class spirit. The society is confident that it can invest indefinitely at an interest rate of 2.5% a year. How much does the society need to endow its prize? $\boxed{2}$

4.3 Fill in the following amortization schedule for a debt of R9,000 which is to be repaid by five equal quarterly payments with interest at 10% compounded quarterly. If needs be, add or subtract a value from the last payment so that the debt is fully amortized. $\boxed{6}$

Period	Principal Outstanding At Beginning of Period	Interest for Period	Payment at End of Period	Principal Repaid at End of Period
1				
2				
3				
4				
5				
Total				

Question 5 [10 marks]

5.1 A committee of four is to be formed. The committee must have a female student representative, a male student representative, an academic and a member of the university executive. There are two members of the executive available, four female students available, there are four male students available and there are three academics available. In how many ways can the committee be formed? ☐3

5.2 Let $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$ be a sample space for an experiment, and let $E_1 = \{1, 2, 3, 4, 5, 6\}$, $E_2 = \{4, 5, 6, 7\}$ be events. Find

(a) $E_1 \cup E_2$, ☐1

(b) $E_1 \cap E_2$, ☐1

(c) $E_1' \cup E_2$. ☐1

(d) Are the events E_1 and E_2 mutually exclusive? Explain. ☐1

5.3 Three cards are randomly drawn, with replacement, from an ordinary deck of 52 playing cards. Find the probability that the cards drawn, in order, are an ace, a face card (a jack, queen or king), and a spade.   3

Question 6 [14 marks]

6.1 Using limits, find all the asymptotes of $f(x) = \frac{2x^2}{x^2+x-6}$.   5

6.2 Use the formal definition of a derivative to show that if f and g are differentiable functions, then fg is differentiable and

$$\frac{d}{dx}\left(f(x)g(x)\right) = \frac{d}{dx}(f(x))g(x) + f(x)\frac{d}{dx}(g(x))$$

$\boxed{5}$

6.3 Evaluate the derivative of $x^2 e^{2x}$.

$\boxed{2}$

6.4 Let $f(x) = \frac{1}{12}x^4 - \frac{1}{2}x^3 + x^2 - 4$. Find all the points where $f''(x) = 0$.

$\boxed{2}$

Question 7 [8 marks]

Use a financial calculator to answer the following questions.

7.1 Find the effective rate that corresponds to a nominal rate of 5% compounded monthly. ☐1

7.2 How long will it take R1,000 to amount to R10,000 if it is invested at 6% compounded monthly? Express your answer in years rounded to two decimal places. ☐1

7.3 Find the net present value for the following cash flows, which can be purchased by an initial investment of R80,000.

Year	Cash Flow
2	R34,000
4	R35,000
5	R36,000

Assume that interest is at 5% compounded semiannually. ☐1

7.4 Find the amount of an annuity due that consists of 13 yearly payments of R1,500.00, if the interest rate is 4% compounded annually. ☐1

7.5 A car loan of R235,000 is to be amortized over 60 months at an interest rate of 7.2% compounded monthly. Find:

(a) the monthly payment, ☐1

(b) the balance outstanding after the 59th payment, ☐1

(c) the finance charge. ☐1

7.6 The management of a corporation anticipates a capital expenditure of R5, 000, 000 in 5 years for the purpose of purchasing replacement machinery. To finance this purchase, a sinking fund that earns interest at the rate of 10% per annum compounded quarterly will be set up. Determine the amount of each (equal) quarterly installment that should be deposited in the fund. (Assume that the payments are made at the end of each quarter.)   1