University of Johannesburg

Faculty of Science

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS	
MODULE	**MAT3A20/MAT02A3** DISCRETE MATHEMATICS
CAMPUS	**APK**
SUPPLEMENTARY EXAM	**JULY 2019**

Date 07/2019

Examiner	Dr W Morton
External Examiner	Dr R Kellerman
Duration 150 Minutes	70 Marks

Surname and initials: ____________________

Student number: ____________________

Tel No.: ____________________

INSTRUCTIONS:

1. The paper consists of **12** printed pages, **excluding** the front page.
2. Read the questions carefully and answer all questions.
3. **Write out all calculations (steps) and motivate all answers.**
4. Questions are to be answered on the question paper in the space provided. Please indicate when the blank side of a page is used.
5. Non-programmable calculators are allowed.

Question 1 [4]

Let $A = \{\{\emptyset\}, \{1\}\}$ and $B = \{\{1, 2\}\}$. Complete the following:

(a) $A \cup B =$________________________________(1)

(b) $\mathscr{P}(A) =$________________________________(1)

(c) $B \times A =$________________________________(1)

(d) $A - (A - B) =$________________________________(1)

Question 2 [4]

Let S be the set of all finite strings of a's and b's (i.e., $S = \{a, b, aa, ab, ba, bb, aaa, ...\}$) and define $C_a : S \to S$ as follows, for $s \in S$,

$$C_a(s) = as \qquad \text{(eg. } C_a(ab) = aab\text{)}.$$

(a) Is C_a injective? Explain. (1)

(b) Is C_a surjective? Explain. (1)

(c) Explain why C_a^{-1} exists and find $C_a^{-1}(abab)$ (1)

(d) Is the composition of C_a^{-1} and C_a, i.e., $C_a^{-1}C_a$, defined? Explain. (1)

Question 3 [4]

Let $\equiv_3 \subseteq \mathbb{Z} \times \mathbb{Z}$ be the relation of having the same remainder after division with 3.

(a) Is $[11]_{\equiv_3} = [13]_{\equiv_3}$? Explain. (1)

(b) Complete: $\mathbb{Z}/_{\equiv_3} =$ ______________________________ (1)

(c) Show that $\equiv_3$ is euclidean. (2)

Question 4 [4]

Let A and B be sets. Prove that $\mathscr{P}(A) \cup \mathscr{P}(B) \subseteq \mathscr{P}(A \cup B)$, but that it is not in general the case that $\mathscr{P}(A) \cup \mathscr{P}(B) = \mathscr{P}(A \cup B)$.

Question 5 [4]

(a) Complete the truth-table: (2)

p	q	r	$p \leftrightarrow \neg r$	$\neg p \vee (q \wedge r)$	$\neg(r \to q)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

(b) A set $\mathcal{S}$ of propositional formulas is said to be **satisfiable** if, and only if, there is some assignment of truth values to the propositional variables that satisfies all the formulas belonging to $\mathcal{S}$. Is $\{p \leftrightarrow \neg r, \neg p \vee (q \wedge r), \neg(r \to q)\}$ satisfiable? Motivate. (2)

Question 6 [2]

Consider the following conditional statement: The continuity of a function is sufficient for it to be differentiable.

(a) Rephrase the statement in the form "__ only if __". (1)

(b) State the inverse of the implication. (1)

Question 7 [4]

Determine whether or not the following logical consequence is valid using a semantic tableaux.

$$(p \vee q) \to (p \wedge q) \models (p \leftrightarrow q).$$

Question 8 [3]

Make use of known equivalences to construct a formula in disjunctive normal form (DNF) equivalent to the following formula:

$$\neg(\neg p \leftrightarrow q) \to (r \vee \neg q).$$

Question 9 [4]

(a) How many permutations of the 26 alphabet letters are there? (1)

(b) How many permutation of the 26 alphabet letters are there that contain none of the sequences MATHS, IS, or FUN? (3)

Question 10 [2]

It is required to select, from the set of numbers $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, a subset of four that will contain either 1 or 2, but not both. How many selections are possible?

Question 11 [4]

A restaurant offers 10 different wines, 4 of which are red, and 8 main course meals, 3 of which involves red meat.

(a) How many ways are there to pair a wine with a main course if a red wine must be paired with a main course containing red meat and a red meat main course must be paired with a red wine? (2)

(b) Now suppose there are no restrictions on the pairing of wines with main course meals. If you visit the restaurant seven times a month every month of the year, is it possible to order a different pairing with each visit? Explain. (2)

Question 12 [6]

Consider the recurrence relation $a_n = 4a_{n-1} + 6$ for $n > 0$ with initial conditions $a_0 = 1$. Solve the recurrence relation using generating functions.

Question 13 [7]

(a) Draw the complete bipartite graph $K_{3,3}$. (1)

(b) $\chi(K_{3,3}) =$ ____________________ (1)

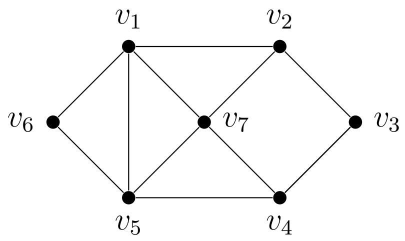
(c) $\Delta(K_{3,3}) =$ ____________________ (1)

(d) Show that $K_{3,3}$ is Hamiltonian? (1)

(e) Prove that $K_{3,3}$ is not planar. (3)

Question 14 [5]

Consider the graph G depicted below and answer the questions that follow:

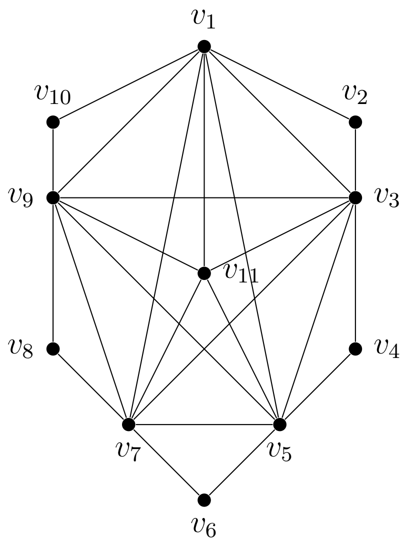


(a) Is G Eulerian? If so, find an Eulerian circuit. If not, explain why not. (3)

(b) Is G bipartite? If so, give the two partite sets; if not, explain why not. (2)

Question 15 [3]

Let H be the graph depicted below.



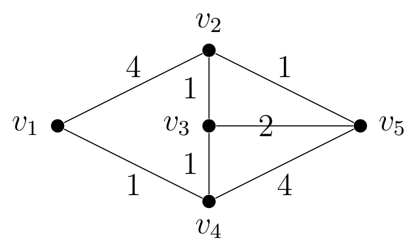
Determine whether or not the graph H is planar. If yes, draw it as a plane graph. If not, use Kuratwoski's theorem to prove that it is not.

Question 16 [2]

A graph G is said to be **outerplanar** if G can be drawn as a plane graph so that all of its vertices lie on the border of its external face. Is K_4 outerplanar? Motivate.

Question 17 [3]

Use Dijkstra's algorithm to calculate the shortest path from v_1 to v_5 in the following graph:



Question 18 [5]

Prove that for every tree T it is the case that

$$n(T) = m(T) + 1.$$