DEPARTMENT OF MATHEMATICS

COURSE: MAT2A20

EXAMINATION

DATE: AUGUST 2019

TIME: 90 min **MARKS: 52**

Examiner: Dr. E. Joubert
Moderator: Dr. F. Schultz

Student number: ____________________

Surname and initials: ________________________________

Contact tel. number: ____

MARKS	%

1. This paper consists of 7 pages.
2. Answer each question in its allocated space. If necessary, use the back of the page and indicate that clearly.
3. **Only** non-programmable calculators are allowed.
4. You **have** to show your calculations.

QUESTION 1 **[27]**

1.1 Let V be the set of positive, non-zero real numbers, and consider the following addition and scalar multiplication operations on V :

$$u + v = uv \text{ and } ku = u^k.$$

Answer the following questions:

1.1.1 Define what is meant by a vector space W. [2]

1.1.2 Find the zero vector $\overline{0}$ for V, under the given operations. [2]

1.1.3 Using Question 1.1.2, find the vector $-u$, such that $(-u) + u = \overline{0}$ and $u + (-u) = \overline{0}$. [2]

1.1.4 Is V a vector space? Motivate your answer. [4]

1.1.5 If we change the addition operation in V to normal real number addition, will V still be a vector space? Motivate your answer. [2]

1.1.6 Define what is meant by a subspace W of V. [1]

1.1.7. Let W be the set of rational numbers (recall that a rational number x is a number that can be written in the form $\frac{a}{b}$, where $b \neq 0$ and a and b are integers). Is W a subspace of V? Motivate your answer. [2]

1.2 Define what is meant by the null space of a matrix A. [1]

1.3 Show that the solution set of a homogenous system $A\overline{x} = \overline{0}$ in n unknowns is a subspace of R^n. [3]

1.4 Consider the following matrix A, where s is an unknown. Answer the following questions:

$$A = \begin{bmatrix} 1 & 1 & s \\ 1 & s & 1 \\ s & 1 & 1 \end{bmatrix},$$

1.4.1 Find the values of s for which the null space of A is the origin only. [1]

1.4.2 Find the values of s for which the null space of A will be a line through the origin. [1]

1.4.3 Find the values of s for which the null space of A will be a plane through the origin. [1]

1.4.4 If $s = 1$, find a base for the null space of A, and the dimension of the null space of A. [2]

1.4.5 If $s = 0$, find a basis for the row space of A, and a basis for the column space of A. [3]

QUESTION 2 **[25]**

2.1 Let V be the space spanned by $f_1 = \sin x$ and $f_2 = \cos x$. Answer the following questions:

2.1.1 Define what is meant if we say that $\{f_1, f_2\}$ spans V. [1]

2.1.2 Define what is meant if we say that a set S is a basis for V. [1]

2.1.3 Show that for any value of θ, $g_1 = \sin(x + \theta)$ and $g_2 = \cos(x + \theta)$ are vectors in V. [2]

2.1.4 Can one say that $1 \in span(\{f_1, f_2\})$? Motivate your answer. [2]

2.1.5 Show that $g_1 = 2 \sin x + \cos x$ and $g_2 = 3 \cos x$ form a basis for V. [2]

2.1.6 Find the transition matrix $P_{B \to B'}$, where $B = \{f_1, f_2\}$ and $B' = \{g_1, g_2\}$. [2]

2.1.7 Find the transition matrix $P_{B' \to B}$. [2]

2.1.8 Compute the coordinate vector $[h]_B$, where $h = 2\sin x - 5\cos x$. Using Question 2.1.4, compute $[h]_{B'}$. [2]

2.2 State and prove the Plus / Minus Theorem. [4]

2.3 Consider the set $S = \{1, 1+x, 2-x, -2+4x, x\}$ of polynomials. Answer the following Questions:
2.3.1 Explain why S cannot be a basis for P_2. [2]

2.3.2 Find span(S) in P_3. [3]

2.3.3 Extend the set $S' = \{x+1, 2-x\}$ to be a base for P_3. [2]