University of Johannesburg

Faculty of Science

DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

MODULE **MAT3A01 / MAT01A3**
REAL ANALYSIS

CAMPUS **APK**

EXAM **MAY 2019**

Date 23/05/2019 — Session 08:30 – 11:30

Assessor — Dr G Braatvedt

External Moderator — Dr L Lindeboom

Duration 3 Hours — 50 Marks

Surname and initials:

Student number:

Tel No.:

INSTRUCTIONS:

1. The paper consists of **11** printed pages, **excluding** the front page.
2. Read the questions carefully and answer all questions.
3. **Write out all calculations (steps) and motivate all answers.**
4. Questions are to be answered on the question paper in the space provided. Please indicate when the blank side of a page is used.
5. **Good luck - write well :-)**

Question 1 [4]

State the following theorems:

(a) The *Bolzano-Weierstrass Theorem.* (2)

(b) The *Location of Roots Theorem.* (2)

Question 2 [4]

True or false (if true: give a short motivation; if false, give a counterexample):

(a) If $(I_n) = ([a_n, b))$ is a nested sequence of bounded intervals, then we know that there exists $x \in \bigcap_{n=1}^{\infty} I_n$. (2)

(b) If f is continuous on $A \subseteq \mathbb{R}$ then f is uniformly continuous on A. (2)

Question 3 [4]

Prove that $\mathbb{R}$ is uncountable.

Question 4 [5]

Determine $\inf S$ and $\sup S$ in full detail if $S = \left\{ \frac{n-m^2}{n+m} : n, m \in \mathbb{N} \right\}$.

Question 5 [5]

State and prove the *Monotone Convergence Theorem* for decreasing sequences.

Question 6 [4]

Establish the convergence or divergence of the following sequences in full detail:

(a) $\left(\frac{n!}{n+1}\right)$ (2)

(b) $\left(\frac{3n+4}{2n+1}\right)$ (2)

Question 7 [4]

Prove or disprove that (x_n) is Cauchy, using the definition of Cauchy, if

$$x_n = \left(\frac{(-1)^n(1-\sqrt{n})}{\sqrt{n}+1} \right).$$

Question 8 [4]

Consider the function $f(x) = \frac{1}{x(x+2)}$.
Show, using the definition of continuity, that f is continuous on $(-2, 0)$.

Question 9 [6]

Let $f(x) = \frac{1}{\sqrt{x}}$.

(a) State the *Continuous Extension Theorem.* (2)

(b) Hence, establish whether or not f is uniformly continuous on $(0, 3]$, in full detail. (3)

(c) Is f Lipschitz on $(0, 3]$? Explain. (1)

Question 10 [5]

Prove the theorem: let $I = [a, b]$ be a closed bounded interval and $f : I \to \mathbb{R}$ continuous on I. Then f is bounded on I.

Question 11 [5]

Consider the function $f(x) = \frac{1}{x-1}$.

(a) Determine whether or not f is continuous on $(1, \infty)$. (2)

(b) Show that f is not uniformly continuous on $(1, \infty)$, by making use of the *Nonuniform Continuity Criterion*. (3)