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DEPAR	TMENT OF PURE	AND APPLIED MATHEMATICS
MODULE	MAT01A2 / MAT2A10 Sequences, Series and Vector Calculus	
CAMPUS	APK	
ASSESSMENT	SUPPLEMENTARY EXAMINATION	
DATE		JULY 2019
ASSESSOR(S)		M. SIAS
INTERNAL MODERATOR		C. MARAIS
DURATION: 120 MINUTES		MARKS: 50
SURNAME AND	INITIALS	
STUDENT NUME	3ER	
CONTACT NUM	3ER	
NUMBER OF PA	GES: $1 + 13$ PAGES	6
INSTRUCTIONS:		
1. ANSWER ALL	THE QUESTIONS O	N THE PAPER IN PEN.
2. NO CALCULA	TORS ARE ALLOWE	D.
3. SHOW ALL CA	ALCULATIONS AND	MOTIVATE ALL ANSWERS.
4. IF YOU REQU INDICATE TH		CONTINUE ON THE <u>FACING</u> BLANK PAGE AND

Question 1

For questions (1.1) - (1.5), please circle only **ONE** correct answer:

(1.1) Given the series:

$$A := \sum_{m=1}^{\infty} \frac{(-1)^m}{m^{0.1}}$$
 and $B := \sum_{m=1}^{\infty} (-1)^m \frac{2^m}{m^{100}}.$

 $\left[5\right]$

(1)

Determine whether each series is convergent or divergent.

- (a) A is convergent, B is divergent.
- (b) A is divergent, B is convergent.
- (c) The series are both convergent.
- (d) The series are both divergent.

(1.2) If a series $\sum a_n$ is conditionally convergent, then $\lim_{n\to\infty} |a_n| \neq 0$. (1)

(a) True (b) False

(1.3) The Root Test can be used to determine whether the series $\sum e^n \left(1 + \frac{1}{n}\right)^{-n^2}$ converges. (1)

- (a) True (b) False
- (1.4) Suppose that the series $\sum_{n=0}^{\infty} c_n (x-2)^n$ converges when x = 4 and diverges when x = -4. What can be said about the convergence or divergence of the following series: (1)

$$C := \sum_{n=0}^{\infty} c_n (-1)^n$$
 and $D := \sum_{n=0}^{\infty} c_n 7^n$.

- (a) C is convergent, D is divergent.
- (b) C is divergent, D is convergent.
- (c) The series are both convergent.
- (d) The series are both divergent.

(1.5) A power series representation and radius of convergence for $f(x) = \frac{1}{4+x^2}$ is: (1)

- (a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{4^{n+1}}$; R = 1. (b) $\sum_{n=0}^{\infty} \frac{x^{2n}}{4^n}$; R = 1.
- (c) $\sum_{n=0}^{\infty} \frac{x^{2n}}{4^{n+1}}$; R = 2.
- (d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{4^{n+1}}$; R = 2.

Question 2

Show that the $\lim_{n\to\infty} \frac{3n^2+1}{4n^2+1} = \frac{3}{4}$ by using the precise definition of a limit of a sequence.

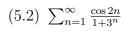
[4]

Question 3 Give an example of two divergent sequences $\{a_n\}$ and $\{b_n\}$ such that the sequence $\{a_nb_n\}$ is [2]convergent.

Question 4 Let $a_1 = \sqrt{3}$ and let $a_{n+1} = \sqrt{3a_n}$ for $n \in \mathbb{N}$. Show that $\{a_n\}$ is increasing and bounded above by 3, and find its limit. [4]

 $\frac{\textbf{Question 5}}{\text{Test the following series for convergence or divergence:}}$

(5.1)
$$\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^n}{n^{3n}}$$
 (3)



(2)

[5]

 $\frac{\textbf{Question 6}}{\text{State and prove the Alternating Series Test.}}$

 $\frac{\textbf{Question 7}}{Find a Maclaurin series for the given function:}$

$$f(x) = \begin{cases} \frac{1 - \cos x}{x^2} & \text{if } x \neq 0\\ \\ \frac{1}{2} & \text{if } x = 0. \end{cases}$$

 $\frac{\textbf{Question 8}}{\textbf{Use the binomial series series to expand } f \text{ and state its radius of convergence:}}$

$$f(x) = (1-x)^{\frac{2}{3}}$$
.

[4]

Question 9
Determine
$$\mathbf{r}(t)$$
 if $\mathbf{r}'(t) = \left\langle \frac{1}{1+t^2}, \cos^2 t, te^{t^2} \right\rangle$ and $\mathbf{r}(0) = \langle 1, 0, 1 \rangle$.

[3]

Question 10

Reparametrize the curve with respect to arc length measured from the point where t = 0 in the direction of increasing t: [4]

$$\mathbf{r}(t) = \langle 2\sin t, 4, 2\cos t \rangle.$$

Question 12 Show that if there is a $c \in \mathbb{R}$ such that $|\mathbf{r}(t)| = c$ for all t, then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$. [2]

Question 13 Prove that the curvature of a curve C with vector function $\mathbf{r}(t)$ is given by the following formula:[4]

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

 $\frac{\textbf{Question 14}}{\textbf{A particle moves with position function}}$

$$\mathbf{r}(t) = \left\langle t^3, 1 - t^2, t + 7 \right\rangle.$$

Determine the normal component of the accelaration of the particle.

[4]