UNIVERSITY OF JOHANNESBURG

PROGRAM	: NATIONAL DIPLOMA *INDUSTRIAL ENGINEERING*
SUBJECT	: **OPERATIONAL RESEARCH III**
CODE	: **BOA321**
DATE	: MAIN EXAMINATION 26 NOVEMBER 2019
WEIGHT	: 40:60
DURATION	: (SESSION 2) 12:30-15:30 HOURS
TOTAL MARKS	100

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NUMBER OF PAGES	: 8 PAGES

INSTRUCTIONS

1. ANSWER ALL QUESTIONS.
2. WRITE LEGIBLY AND NUMBER YOUR ANSWERS ACCORDING TO THE QUESTION PAPER.
3. ONE CALCULATOR PERMITTED PER STUDENT.

QUESTION 1

1.1 An urn contains 8 black chips, 10 yellow chips and 2 blue chips. A chip is drawn and replaced and then a second chip drawn. What is the probability of:

1.1.1 A blue chip on the first draw. (2)

1.1.2 A blue chip on the first draw and a black chip on the second draw. (3)

1.1.3 Two yellow chips being drawn. (3)

1.1.4 A black chip on the second given that a blue chip was drawn on the first. (2)

1.2 A local company produces cases for personal computers and other electronic equipment. The quality control inspection procedure is to select 6 items, and if there are 0 or 1 defectives cases in the group of 6, the process is said to be in control. If the number of defects is more than 1, the process is out of control. Suppose that the true proportion of defectives items is 0.15.

What is the probability that there will be 0 or 1 defects in a sample of 6. (8)

[18]

QUESTION 2

Mary Baloyi has been thinking about starting her own independent gasoline station. Mary's problem is to decide how large her station should be. The annual returns will depend on both the size of her station and a number of marketing factors related to the oil industry and demand for gasoline. After a careful analysis, Mary developed the following table:

The values in the table are in 1000s

Size of station	Good market (R in 1000s)	Fair market (R in 1000s)	Poor market (R in 1000s)
Small	50	20	-10
Medium	80	30	-20
Large	100	30	-40
Very large	300	25	-160

2.1 Construct a decision table for this problem. (10)

2.2 What is the maximax decision? (2)

2.3 What is the maximin decision? (2)

2.4 What is the equally likely decision? (2)

2.5 What is the criterion of realism decision? Use a α value of 0.8. (2)

2.6 Develop an opportunity loss table. (5)

2.7 What is the minimax regret decision? (2)

[25]

QUESTION 3

Students in operational research class have just received their grades on the first test. The instructor has provided information about the first test grades in some previous classes as well as the final averages for the same students. Some of these grades have been sampled and are as follows:

Student	1	2	3	4	5	6	7	8	9
1st test grade	98	77	88	80	96	61	66	95	69
Final average	93	78	84	73	84	64	64	95	76

3.1 Develop a regression model that could be used to predict the final average in the course based on the first test grade. (23)

3.2 Predict the final average of a student who made an 83 on the first test. (2)

[25]

QUESTION 4

Consulting income at Morrison Associates for period February to July are as follows:

Month	Income (R 1000s)
February	70.0
March	68.5
April	64.8
May	71.7
June	71.3
July	72.8

4.1 Assume that the initial forecast for February is R65 000 and the smoothing constant selected is $\alpha = 0.3$, use exponential smoothing to forecast August's income. (12)

4.2 Compute the MAD for this model. (6)

[18]

QUESTION 5

Amos Banda is thinking about producing a new type of electric razor for men. If the market is favorable he can get a return of R100 00, but if the market for this new type of razor is unfavorable he can lose R60 000. Since, Adam Sithole is a good friend of Amos, Adam is considering the possibility of using a market research to gather additional information about the market for the razor. Adam has suggested that Amos use either a survey or a pilot study to test the market. The survey will cost R5000 and the pilot study will cost R20 000. Adam has suggested that it would be a good idea for Amos to conduct either the survey or the pilot study before he makes the decision whether to produce the new razor. However, Amos is not sure if the value of the survey or the pilot study is worth the cost.

Draw the decision tree for this problem. (14)

[14]

TOTAL MARKS: 100

FORMULA SHEET

A basic statement of probability

$0 \leq P\ (\text{event}) \leq 1$

$$P\ (\text{event}) = \frac{Number\ of\ occurences\ of\ event}{Total\ number\ of\ trials\ or\ outcomes}$$

Law of addition for mutually exclusive events

P (A or B) = P (A) + P (B)

Law of addition for events that are not mutually exclusive events

P (A or B) = P (A) + P (B) – P (A and B)

Joint probability for independent events

P (AB) = P (A) x P (B)

Conditional probability

$$P\ (A|B) = \frac{P\ (AB)}{P\ (B)}$$

Bayes' Theorem in general form

$$P\ (A|B) = \frac{P(\ B|A)\text{P (A)}}{P(B|A)P(A) + P(B|A')P(A')}$$

Expected value of a discrete probability distribution

$$\mathbf{E(X)} = \sum_{i=1}^{n} XiP(Xi)$$

Variance of a discrete probability distribution

$$\text{Variance} = \sum_{i=1}^{n} [X_i - E(X)]^2 P(X_i)$$

Standard deviation of a discrete probability distribution

$\sigma = \sqrt{variance}$

Probability of r successes in n trials (probabilities for the binomial probability distribution)

P (event) = $\dfrac{n!}{r!(n-r)!} p^r q^{n-r}$

An equation that computes the number of standard deviations, Z, the point X is from the mean μ.

$Z = \dfrac{X - \mu}{\sigma}$

Expected value of perfect information (EVPI)

EVPI = expected value with information – maximum EMV

Where EVwPI = (best payoff for the 1^{st} nature x probability of the first) + (best payoff for the 2^{snd} nature x probability of the second nature) + ….. + (best payoff for the last nature x probability of the last nature)

Expected Monetary Value

EMV = (best payoff for the 1^{st} nature x probability of the first) + (best payoff for the 2^{nd} nature x probability of the second nature) + ….. + (best payoff for the last nature x probability of the last nature)

Underlying linear model for simple linear regression

$Y = \beta_0 + \beta_1 X + \varepsilon$

Simple liner regression model computed from a sample

$\hat{Y} = b_0 + b_1 X$

Error in regression model

$$e = Y - \hat{Y}$$

Formulas for simple linear regression

$$\overline{X} = \frac{\sum X}{n} = \text{average (mean) of } X \text{ values}$$

$$\overline{Y} = \frac{\sum Y}{n} = \text{average (mean) of } Y \text{ values}$$

$$b_1 = \frac{\sum (X - \overline{X})(Y - \overline{Y})}{\sum (X - \overline{X})^2}$$

$$b_0 = \overline{Y} - b_1 \overline{X}$$

Intercept in the regression line

$$\text{SST} = \sum (Y - \overline{Y})^2$$

Total sum of squares

$$\text{SSE} = \sum e^2 = \sum (Y - \hat{Y})^2$$

Sum of squares due to error

$$\text{SSR} = \sum (\hat{Y} - \overline{Y})^2$$

Sum of squares due to regression

$$\text{SST} = \text{SSR} + \text{SSE}$$

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n}$$

$$\text{MSE} = \frac{\sum (\text{error})^2}{n}$$

$$\text{MAPE} = \frac{\sum \left| \frac{\text{error}}{\text{actual}} \right| 100\%}{n}$$

Moving average forecast

$$F_{t+1} = \frac{Y_t + Y_{t-1} + \ldots + Y_{t-n+1}}{n}$$

Weighted moving average forecast

$$F_{t+1} = \frac{w_1 Y_t + w_2 Y_{t-1} + \ldots + w_n Y_{t-n+1}}{w_1 + w_2 + \ldots + w_n}$$

Exponential smoothing

$$F_{t+1} = F_t + \alpha(Y_t - F_t)$$

$$\text{Tracking signal} = \frac{\text{RSFE}}{\text{MAD}} = \frac{\sum(\text{forecast error})}{\text{MAD}}$$

λ = mean number of arrivals per time period
μ = mean number of customers or units served per time period

The average number of customers or units in the system, ***L***

$$L = \frac{\lambda}{\mu - \lambda}$$

The average time a customer spends in the system, ***W***

$$W = \frac{1}{\mu - \lambda}$$

The average number of customers in the queue, L_q

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

The average time a customer spends waiting in the queue, W_q

$$W_q = \frac{\square\lambda}{\mu(\mu - \lambda)}$$

The utilization factor for the system, ρ**, the probability the service facility is being used**

$$\rho = \frac{\lambda}{\mu}$$

The percent idle time, P_0**, or the probability no one is in the system**

$$P_0 = 1 - \frac{\lambda}{\mu}$$

The probability that the number of customers in the system is greater than ***k***, $P_{n>k}$

$$P_{n>k} = \left(\frac{\lambda}{\mu}\right)^{k+1}$$

Total service cost = mC_s

Where:

m = number of channels
C_s = service cost (labor cost) of each channel

Total waiting cost based on time in the system = $(\lambda W)C_w$