UNIVERSITY OF JOHANNESBURG

PROGRAM : NATIONAL DIPLOMA
ENGINEERING: MECHANICAL

SUBJECT : **APPLIED STRENGTH OF MATERIALS III**

CODE : **ASM 301**

DATE : SUMMER EXAMINATION 2019
18 NOVEMBER 2019

DURATION : (X-PAPER) 08:30-11:30

WEIGHT : 40 : 60

TOTAL MARKS : (100 marks = 100%)

EXAMINER : Dr K. TEKWEME

MODERATOR : Dr H. NGWANGWA

NUMBER OF PAGES : 4 PAGES + 1 ANNEXURE

INSTRUCTIONS:

1) ALL SKETCHES MUST BE DONE WITH DRAWING INSTRUMENTS. MARKS WILL BE DEDUCTED FOR UNTIDY WORK.
2) STUDENTS MUST SUPPLY OWN DRAWING EQUIPMENT.
3) ANSWER ALL THE QUESTIONS.

QUESTION 1

A steel leaf spring carries a central load of 4 kN. The plates are to be made of 12 plates 6 cm wide and 5 mm thick. The bending stress is not to exceed 200 N/ mm^2. Determine:

1.1 the length of the spring; (2)

1.2 the deflection produced at the centre of the spring; and (2)

1.3 the radius of curvature of leaves. (2)

E= 2 x 10^5 N/ mm^2 .

[6]

QUESTION 2

An overhanging beam carries the loads as shown in Figure 1. Calculate:

2.1 the support reactions; (2)

2.2 the slope at A; (6)

2.3 the deflection at mid-span; (4)

2.4 the deflection at C; and (2)

2.5 the slope at 5 m from the left end of the beam. (2)

E= 200 GPa and I= 180 x 10^{-6} m^4.

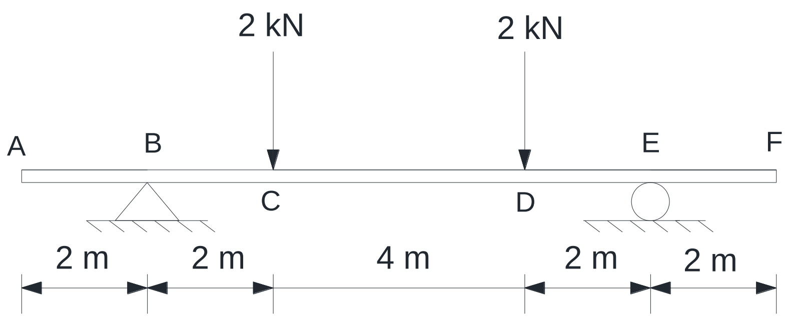


Figure 1

[16]

QUESTION 3

A column of steel is 10 m long and has a cross-section of 120 mm x 180 mm. Use a factor of safety of 3 and calculate the Euler buckling load and the safe load for the column assuming:

3.1 both ends are fixed; (9)

3.2 both ends are ball-jointed; and. (4)

3.3 one end is fixed and the other end is free. (4)

$E = 2 \times 10^5$ N/mm^2

[17]

QUESTION 4

A plane element is subjected to the stresses shown in Figure 2. Determine:

4.1 the resultant stress on plane AB using formulae for transformation of stress; (5)

4.2 the resultant stress on plane AB from first principles; and (9)

4.3 confirm your answers using Möhrs' stress circle, and clearly indicate plane AB. (6)

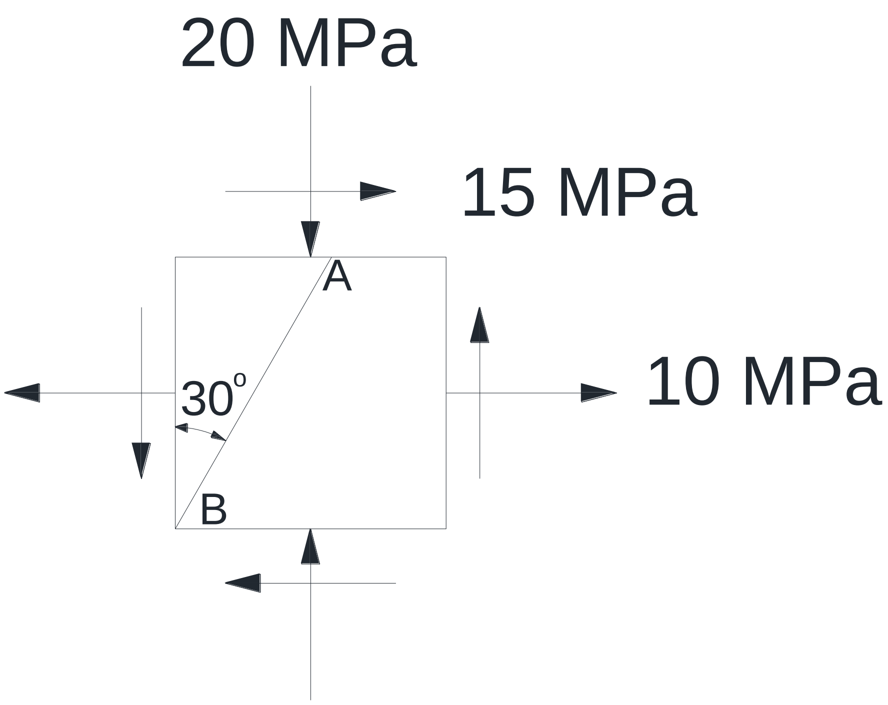


Figure 2

[20]

QUESTION 5

At a certain point on a material, the following strains were measured:

$\varepsilon_x = 320 \times 10^{-6}$; $\varepsilon_y = -150 \times 10^{-6}$; $\gamma_{xy} = 700 \times 10^{-6}$

If E = 200 GPa and ν = 0.28, calculate:

5.1 the linear and shear strains on a plane inclined at an angle 40^{o} anticlockwise form the positive x-axis; (7)

5.2 the principal strains and maximum shear strain; (6)

5.3 use Möhr's strain circle to verify the answers obtained for 5.1 and 5.2; and (4)

5.4 the principal stresses and their directions. (11)

[28]

QUESTION 6

Calculate the hoop and radial stresses at the inner, mean and outer radii across the section of a thick cylinder 250 mm internal diameter and 100 mm thick when the cylinder is subjected to an internal pressure of 22 MPa.

Take E = 203 GPa and ν = 0.3.

[6]

QUESTION 7

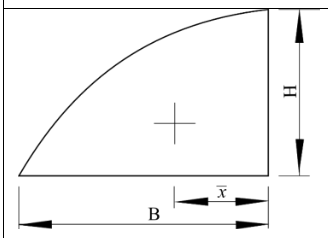
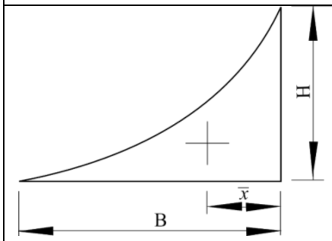
The yield strength of a material in a ductile state is 305 MPa. Use both of the theories of elastic failure, determine the factor of safety for the following principal stress values at a point of a ductile material:

76 MPa (T), 90 MPa (T) and 120 MPa (C).

[7]

TOTAL MARKS AVAILABLE = 100 (100 MARKS = 100%)

FORMULAE SHEET

<table>
<tr><td>1. Semi-elliptical leaf-springs</td><td>$\sigma = \frac{3WL}{2bnt^2}$
$\delta = \frac{3WL^3}{8bEnt^3}$
$\delta = \frac{l^2}{8R}$</td></tr>
<tr><td>2. Quarter elliptical leaf-springs</td><td>$\sigma = \frac{6WL}{bnt^2}$
$\delta = \frac{6WL^3}{bEnt^3}$</td></tr>
<tr><td>3. Deflection of beams</td><td>$EI\frac{d^2y}{dy^2} = M$</td></tr>
<tr><td>
</td><td>$\bar{x} = \frac{3}{8}B$
$Area = \frac{2}{3}$BH</td></tr>
<tr><td>
</td><td>$\bar{x} = \frac{1}{4}B$
$Area = \frac{1}{3}$BH</td></tr>
<tr><td>4. Buckling of Struts</td><td>Euler Buckling: $P_E = \frac{\pi^2 EI}{L_e^2}$
Rankine Buckling: $P_R = \frac{S_y A}{\left[1+a\left(\frac{L_e}{K}\right)^2\right]}$
Validity Limit: $\left(\frac{L_e}{K}\right)_{lim} = \sqrt{\frac{2\pi^2 E}{S_y}}$
Both ends pin-jointed: $l_e = l$
Both ends fixed: $l_e = l/2$
One end fixed and the other end free: $l_e = 2l$
One end fixed and the other end pinned: $l_e = l/\sqrt{2}$
$I = \frac{bh^3}{12}$</td></tr>
</table>

	$k = \sqrt{\frac{I}{A}}$
5. Transformation of Stress	Direct and shear plane stresses on an oblique plane θ degrees (anticlockwise) from the vertical axis: $\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)Cos2\theta + \tau_{xy}Sin2\theta$ $\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)Sin2\theta - \tau_{xy}Cos2\theta$ Maximum principal direct stresses: $\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ Maximum principal shear stress: $\tau_{max} = \pm\frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \pm\frac{1}{2}(\sigma_1 - \sigma_2)$ Direction of maximum principals stresses: $tan\ 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$
6. Analysis of Strain	Bi-axial strain: $\varepsilon_x = \frac{\Delta x}{x} = \frac{\sigma_x}{E} - \frac{\upsilon\sigma_y}{E}$; $\sigma_x = \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y)$ $\varepsilon_y = \frac{\Delta y}{y} = \frac{\sigma_y}{E} - \frac{\upsilon\sigma_x}{E}$; $\sigma_y = \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x)$ $\varepsilon_A = \frac{\Delta A}{A} = \varepsilon_y + \varepsilon_x$ Tri-axial volumetric strain: $\varepsilon_x = \frac{\Delta x}{x} = \frac{\sigma_x}{E} - \frac{\upsilon\sigma_y}{E} - \frac{\upsilon\sigma_z}{E}$ $\varepsilon_y = \frac{\Delta y}{y} = \frac{\sigma_y}{E} - \frac{\upsilon\sigma_x}{E} - \frac{\upsilon\sigma_z}{E}$ $\varepsilon_z = \frac{\Delta z}{z} = \frac{\sigma_z}{E} - \frac{\upsilon\sigma_x}{E} - \frac{\upsilon\sigma_y}{E}$ $\varepsilon_v = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\nu)$

Strain in circular shafts:

$$\varepsilon_L = \frac{\Delta L}{L} = \frac{1}{E}(\sigma_L - 2\upsilon\sigma_D)$$

$$\varepsilon_D = \frac{\Delta \mathrm{D}}{D} = \frac{1}{E}(\sigma_D - \upsilon\sigma_D - \upsilon\sigma_L)$$

$$\varepsilon_v = \frac{\Delta V}{V} = \varepsilon_L + 2\varepsilon_D$$

Strain in thin cylinders:

$$\varepsilon_L = \frac{\Delta L}{L} = \frac{\sigma_L}{E} - \frac{\upsilon\sigma_H}{E} = \frac{pd}{4tE}(1 - 2\upsilon)$$

$$\varepsilon_H = \frac{\Delta H}{H} = \frac{\sigma_H}{E} - \frac{\upsilon\sigma_L}{E} = \frac{pd}{4tE}(2 - \upsilon)$$

$$\varepsilon_v = \varepsilon_L + 2\varepsilon_H = \frac{pd}{4tE}(5 - 4\upsilon)$$

Strain in thin spheres:

$$\varepsilon_H = \frac{\Delta H}{H} = \frac{1}{E}(\sigma_H - \upsilon\sigma_H) = \frac{pd}{4tE}(1 - \upsilon)$$

$$\varepsilon_v = 3\varepsilon_H = \frac{3pd}{4tE}(1 - \upsilon)$$

Elastic constants:

$E = 2G(1 + \upsilon);\ E = 3K(1 - 2\upsilon)$

Direct and shear plane strains on an oblique plane θ degrees (anticlockwise) from the vertical axis:

$$\varepsilon_\theta = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)Cos2\theta + \frac{1}{2}\gamma_{xy}Sin2\theta$$

$$\gamma_\theta = -(\varepsilon_x - \varepsilon_y)Sin2\theta + \gamma_{xy}Cos2\theta$$

Maximum principal direct strains:

$$\varepsilon_{1,2} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) \pm \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

Direction of maximum principals strains:

$$tan\ 2\theta = \frac{\gamma_{xy}}{(\varepsilon_x - \varepsilon_y)}$$

Shear strain: $\gamma_{xy} = \frac{\tau_{xy}}{G}$

Maximum shear strain

	$\gamma_{max} = \pm\sqrt{\left(\varepsilon_x - \varepsilon_y\right)^2 + \gamma_{xy}^2}$ Principal stresses $\sigma_1 = \frac{E}{(1-\nu^2)}(\varepsilon_1 + \nu\varepsilon_2)$ $\sigma_2 = \frac{E}{(1-\nu^2)}(\varepsilon_2 + \nu\varepsilon_1)$
7. Thick Cylinders	Radial stress: $\sigma_r = A - \frac{B}{r^2}$ Hoop stress: $\sigma_h = A + \frac{B}{r^2}$ Stresses in thick cylinders due to an internal pressure P_i and external pressure, P_o: $\sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left[1 - \frac{r_o^2}{r^2}\right] - \frac{P_o r_o^2}{r_o^2 - r_i^2}\left[1 - \frac{r_i^2}{r^2}\right]$ $\sigma_h = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left[1 + \frac{r_o^2}{r^2}\right] - \frac{P_o r_o^2}{r_o^2 - r_i^2}\left[1 + \frac{r_i^2}{r^2}\right]$ $\sigma_a = \frac{P_i r_i^2 - P_o r_o^2}{(r_o^2 - r_i^2)}$ Stresses in thick cylinders due to an internal pressure only (P_o= 0): $\sigma_r = \frac{P_i r_i^2}{(r_o^2 - r_i^2)}\left[1 - \frac{r_o^2}{r^2}\right]$ $\sigma_h = \frac{P_i r_i^2}{(r_o^2 - r_i^2)}\left[1 + \frac{r_o^2}{r^2}\right]$ $\sigma_a = \frac{P_i r_i^2}{(r_o^2 - r_i^2)}$ Stresses in thick cylinders due to an external pressure only (P_i= 0): $\sigma_r = \frac{-P_o r_o^2}{(r_o^2 - r_i^2)}\left[1 - \frac{r_i^2}{r^2}\right]$ $\sigma_h = \frac{-P_o r_o^2}{(r_o^2 - r_i^2)}\left[1 + \frac{r_i^2}{r^2}\right]$ $\sigma_a = \frac{-P_o r_o^2}{(r_o^2 - r_i^2)}$

	Diametral shrinkage allowance for compound thick cylinder: $s.a = 2r_{int}\left(\frac{1}{E_O}\left(\sigma_{h,O,int} + \upsilon_O P_{int}\right) - \frac{1}{E_I}\left(\sigma_{h,I,int} + \upsilon_I P_{int}\right)\right)$ Diametral shrinkage allowance for shaft and hub: $s.a = 2r_{int}\left(\frac{1}{E_O}\left(\sigma_{h,O,int} - \upsilon_O P_{int}\right) - \frac{1}{E_I}\left(-P_{int} + \upsilon_I P_{int}\right)\right)$ Torque transmitted by a shrink fit: $T = 2\pi\mu r_{int}^2 L P_{int}$ Frictional force to separate a shrink fit: $F = 2\pi\mu r_{int} L P_{int}$
8. Failure theories	Ductile materials: Failure occurs when: Maximum shear stress (Tresca): $\sigma_1 - \sigma_3 \geq \frac{S_y}{n}$ Maximum shear strain energy (von Mises): $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2\left(\frac{S_y}{n}\right)^2$ Brittle materials: Failure occurs when: Maximum principal stress (Rankine): $\sigma_1 \geq \frac{S_{ut}}{n} (if\ \sigma_1 > 0)\ or\ \sigma_3 \geq -\frac{S_{ut}}{n} (if\ \sigma_3 < 0)$ Modified Mohr: $Quadrant\ 1: \sigma_1 \geq \frac{S_{ut}}{n}$ $Quadrant\ 2: \frac{\sigma_3}{S_{ut}} - \frac{\sigma_1}{S_{uc}} \geq \frac{1}{n}$ $Quadrant\ 3: \sigma_3 \geq -\frac{S_{uc}}{n}$ $Quadrant\ 4: \frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} \geq \frac{1}{n}$